

1. a] Define elementary signals. Discuss different types of basic elementary signals with neat figures.

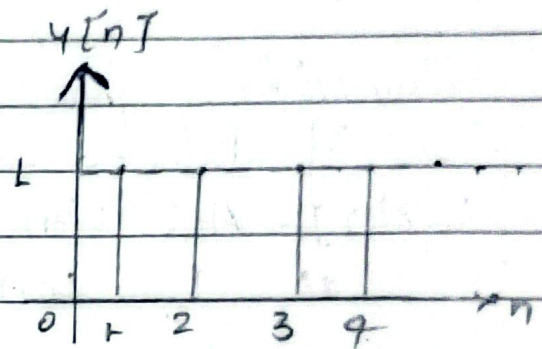
⇒ An elementary signal is a fundamental building block used to represent and analyze more complex signals in systems. They are mathematically simple.

Types :-

i) Unit Step signal.

It is defined as,

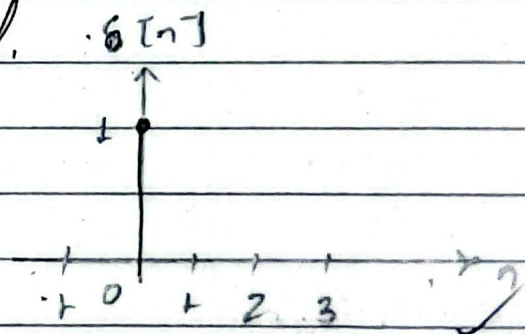
$$u[n] = \begin{cases} 1; & \text{for } n \geq 0 \\ 0; & \text{otherwise} \end{cases}$$



ii) Unit Impulse (Delta) signal.

It is defined as,

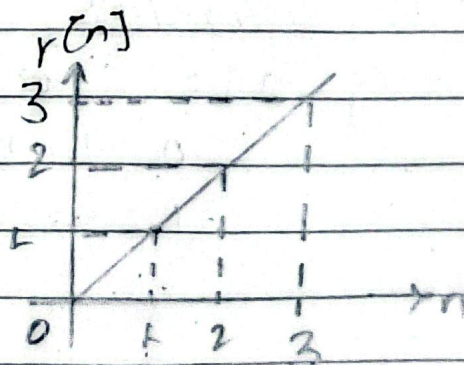
$$\delta[n] = \begin{cases} 1; & \text{for } n = 0 \\ 0; & \text{otherwise} \end{cases}$$



iii) Ramp signal.

It is defined as,

$$r[n] = \begin{cases} n; & \text{for } n \geq 0 \\ 0; & \text{otherwise} \end{cases}$$

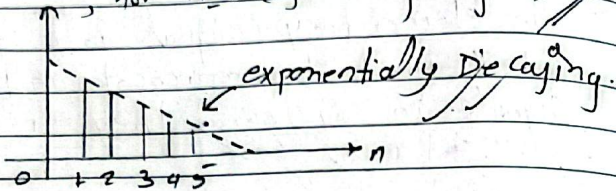


iv) Exponential signals.

It is defined as.

$$x[n] = Ae^{-\alpha n} ; \text{ for } n \geq 0 \text{ (Decaying signal)}$$

$$\text{or } Ae^{\alpha n} ; \text{ for } n \geq 0 \text{ (growing signal)}$$



v) Sinusoidal signal.

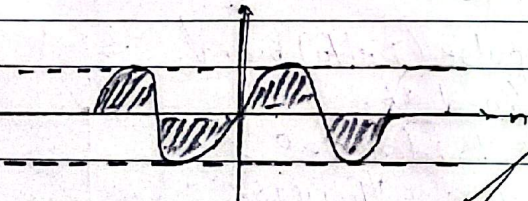
It is defined as.

$$x[n] = A \sin \omega n ; -\infty < n < \infty$$

$$\text{or } A \cos \omega n ; -\infty < n < \infty$$

$$y[n] = \sum$$

$$x[n] = A \sin \omega n$$



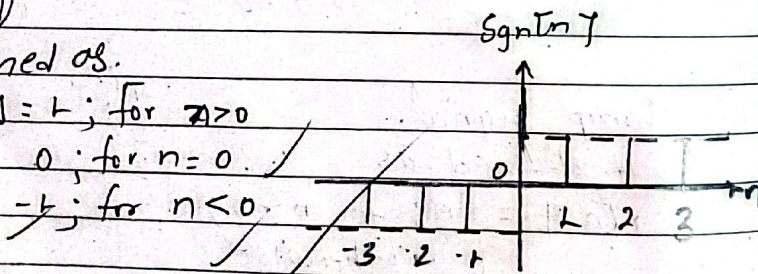
vi) Signum signal

It is defined as.

$$x[n] = \text{sgn}[n] = 1 ; \text{ for } n > 0$$

$$0 ; \text{ for } n = 0$$

$$-1 ; \text{ for } n < 0$$



2.6] Define discrete time system / classify the following system in term of memory, causality, time invariance, linearity and stability.

$$y[n] = 2x[3-n^2]$$

⇒ A discrete time system processes an input sequence $x[n]$ to produce an output sequence $y[n]$, where n represents integer defined time steps.

$$\text{Here, } y[n] = 2x[3-n^2]$$

i) Memory:

A system is memoryless if $y[n]$ depends only on $x[n]$.

Here, $y[n]$ depends on $x[3-n^2]$, not on $x[n]$. So, the system has memory.

ii) Causality:

$$\text{Here, } y[n] = 2x[3-n^2]$$

$$\text{When, } n=0, \quad y[0] = 2x[3-0] = 2x[3]$$

$$n=1, \quad y[1] = 2x[3-1] = 2x[2]$$

$$n=2, \quad y[2] = 2x[3-4] = 2x[-1]$$

Since,

the output $y[n]$ does not depend on present value of input $x[n]$, so, the system is non-causal.

iii) Time invariance: $y[n] = 2x[3-n^2]$ ~~check again.~~
When the i/p is shifted by 'n₀' quantity
 $x[n] \rightarrow x[n-n_0]$ the corresponding
o/p is.

$$y[n, n_0] = 2x[3-(n-n_0)^2] \quad \text{--- (i)}$$

Again,

When o/p is shifted by 'n₀' quantity
 $y[n] \rightarrow y[n-n_0] = 2x[3-(n-n_0)^2] \quad \text{--- (ii)}$

from eqⁿ (i) and (ii)

$y[n-n_0] = y[n, n_0]$, Hence,
the system is ~~in~~variance.

time

iv) Linearity:

$$\alpha \cdot \{ \sum x[n] \} = \{ \sum \alpha \cdot x[n] \}$$

Taking,

$$\alpha \cdot \{ \sum x[n] \} = \alpha \cdot y[n] \\ = \alpha \cdot 2x[3-n^2] \quad \text{--- (i)}$$

$$\text{Again, } \{ \sum \alpha \cdot x[n] \} = 2 \{ \sum \alpha \cdot x[3-n^2] \} \\ = \alpha \cdot 2x[3-n^2] \quad \text{--- (ii)}$$

from (i) and (ii), then

the system is linear.

$$y(n; n_0) = y(n-n_0)$$

$$\alpha \cdot \{ \sum x(n) \} = \{ \sum \alpha \cdot x(n) \}$$

v) Stability:

Let the i/p be bounded, $x[3-n^2] \leq M_x$
then corresponding o/p is,

$$y[n] = |2x[3-n^2]| \\ \leq |2| \cdot M_x < \infty$$

$$\leq 2 \cdot M_x < \infty$$

Since, the o/p is bounded the system is
stable.

or,

$\Rightarrow$ Commutative property:
H state that,

$$y[n] = x[n] \otimes h[n] = h[n] \otimes x[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$

Here, Given,

$$x[n] = \{ 1, 2, 1, 2 \}$$

$$h[n] = \{ 2, 2, -1, 1 \}$$

Now, Convolution,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$= \sum_{k=0}^3 x[k] h[n-k]$$

$$= x[0] h[n] + x[1] h[n-1] + x[2] h[n-2] + x[3] h[n-3] \quad \text{--- (i)}$$

$$y[n] = h[n] + 2h[n-1] + h[n-2] + 2h[n-3] \quad \text{--- (ii)}$$

When,

$$n = -2, y[-2] = h[-2] + 2h[-3] + h[-4] + 2h[-5] = 2$$

$$n = -1, y[-1] = h[-1] + 2h[-2] + h[-3] + 2h[-4] = 2 + 2 \cdot 2 + 0 + 0 = 6$$

$$n = 0, y[0] = h[0] + 2h[-1] + h[-2] + 2h[-3] = -1 + 2 \cdot 2 + 2 = 5$$

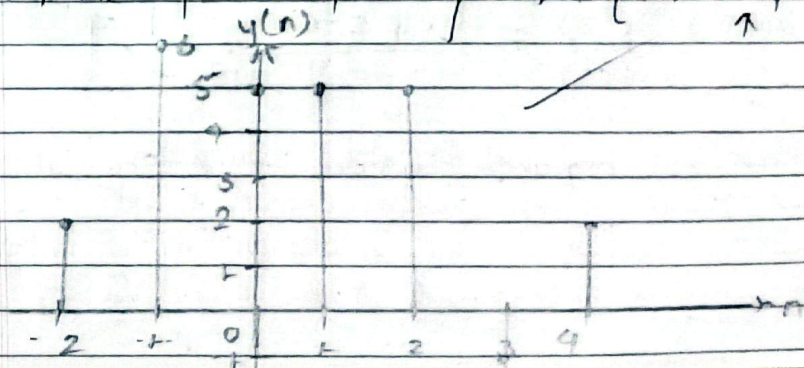
$$n = 1, y[1] = h[1] + 2h[0] + h[-1] + 2h[-2] = 2 + 2 \cdot 1 + 2 + 2 \cdot 2 = 5$$

$$n = 2, y[2] = h[2] + 2h[1] + h[0] + 2h[-1] = 0 + 2 \cdot 1 + (-1) + 2 \cdot 2 = 5$$

$$n = 3, y[3] = h[3] + 2h[2] + h[1] + 2h[0] = 0 + 0 + 1 + 2 \cdot (-1) = -1$$

$$n = 4, y[4] = h[4] + 2h[3] + h[2] + 2h[1] = 0 + 0 + 0 + 2 \cdot 1 = 2$$

So, the required output $y[n] = \{2, 6, 5, 5, 5, -1, 2\}$



$$z = e^{j\omega}$$

2.a) A causal LTI system is described by the difference eqⁿ $y(n) = y(n-1) + y(n-2) + x(n) + 2x(n-1)$. find the system function and frequency response of the system. Plot the poles and zeros and indicate the ROC.

⇒ Here,

$$y(n) = y(n-1) + y(n-2) + x(n) + 2x(n-1)$$

Taking z-transform, on both side,

$$y(z) = z^{-1}y(z) + z^{-2}y(z) + x(z) + 2z^{-1}x(z)$$

$$y(z) - z^{-1}y(z) - z^{-2}y(z) = x(z) + 2z^{-1}x(z)$$

$$y(z)[1 - z^{-1} - z^{-2}] = x(z)[1 + 2z^{-1}]$$

$$\frac{y(z)}{x(z)} = \frac{1 + 2z^{-1}}{1 - z^{-1} - z^{-2}}$$

$$\therefore h(z) = \frac{y(z)}{x(z)} = \frac{1 + 2z^{-1}}{1 - z^{-1} - z^{-2}}$$

Which is required system function.

Now, express in polynomial form
multiply numerator and denominator by z^2

$$H(z) = \frac{z^2 + 2z}{z^2 - z - 1}$$

for, zeros $z^2 + 2z = 0$ ✓
 $z(z+2) = 0$
 $\therefore z = 0, -2$

for, poles, $z^2 - z - 1 = 0$
 using quadratic formula,

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Here, } a=1, b=-1, c=-1$$

So,

$$z = \frac{1 \pm \sqrt{1 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore z_1 = 1.618, z_2 = -0.618$$

ROC, $\rightarrow$ system is causal
 $\hookrightarrow$ ROC is outside the outmost pole.

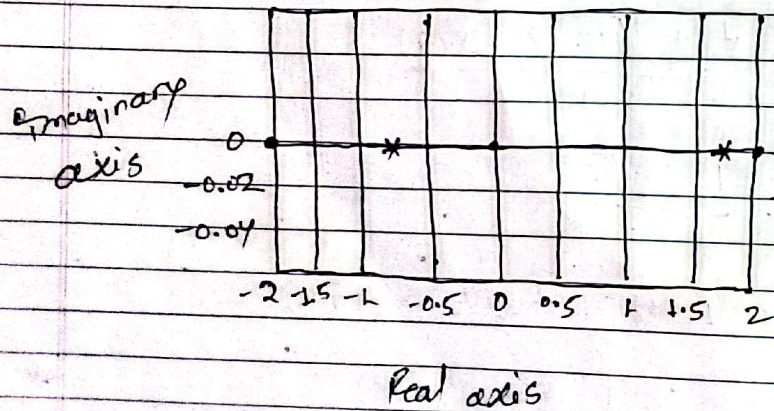
$$\text{ROC: } |z| > 1.618$$

frequency response is obtained by substituting.

So, $z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{1 + 2e^{-j\omega}}{1 - e^{-j\omega} - e^{-2j\omega}}$$

Plot, Pole-zero Plot,



or,

Find the difference eqⁿ satisfying the input & output of an LTI system with transfer function $H(z) = \frac{(1+z^{-1})^2}{(1-0.5z^{-1})(1+0.75z^{-1})}$. Also comment on causality of given transfer function.

$\Rightarrow$ here,

$$H(z) = \frac{(1+z^{-1})^2}{(1-0.5z^{-1})(1+0.75z^{-1})}$$

Numerator,

$$\frac{(1+z^{-1})^2}{= 1 + 2z^{-1} + z^{-2}}$$

Denominator, $(1-0.5z^{-1})(1+0.75z^{-1})$

$$= 1 + 0.75z^{-1} - 0.5z^{-1} - 0.375z^{-2}$$

$$= 1 + 0.25z^{-1} - 0.375z^{-2}$$

$$\text{So, } H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 + 0.25z^{-1} - 0.375z^{-2}}$$

$$\text{or, } Y(z)[1 + 0.25z^{-1} - 0.375z^{-2}] = X(z)[1 + 2z^{-1} + z^{-2}]$$

$$\text{or, } Y(z) + 0.25z^{-1}Y(z) - 0.375z^{-2}Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

Taking inverse z-transform, we get.

$$y(n) + 0.25y(n-1) - 0.375y(n-2) = x(n) + 2x(n-1) + x(n-2)$$

$$y(n) = -0.25y(n-1) + 0.375y(n-2) + x(n) + 2x(n-1) + x(n-2)$$

Which is required different eqⁿ.

Comment on Causality,

- output depends on present and past inputs
- System function contain only negative power of z .
- No future term present

Hence, the system is causal.

Q.6] Find the z-transform and ROC of

$$x(n) = 2\left(\frac{5}{6}\right)^n u(n-1) + 3\left(\frac{1}{2}\right)^{2n} u(n)$$

$$\Rightarrow \text{let } z(n) = x_1(n) + x_2(n)$$

$$x_1(n) = 2\left(\frac{5}{6}\right)^n u(n-1)$$

$$x_2(n) = 3\left(\frac{1}{2}\right)^{2n} u(n)$$

using the property,

$$Z\{a^n u(n-1)\} = \frac{az^{-1}}{1-az^{-1}} \quad |z| > |a|$$

So,

$$X_1(z) = \frac{2 \cdot \frac{5}{6} z^{-1}}{1 - \frac{5}{6} z^{-1}}$$

$$= \frac{\frac{5}{3} z^{-1}}{1 - \frac{5}{6} z^{-1}} \quad |z| > \left|\frac{5}{6}\right|$$

$$X_2(z) = 3\left(\frac{1}{2}\right)^{2n} u(n)$$

$$\text{Since, } \left(\frac{1}{2}\right)^{2n} = \left(\frac{1}{4}\right)^n$$

using the property,

$$Z\{a^n u(n)\} = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

So,

$$X_2(z) = \frac{3 \cdot 1}{1 - \frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{4}$$

Hence,

$$\begin{aligned} X(z) &= X_1(z) + X_2(z) \\ &= \frac{5/3 z^{-1}}{1 - \frac{5}{6}z^{-1}} + \frac{3}{1 - \frac{1}{4}z^{-1}} \end{aligned}$$

Again, Term-1, $|z| > \frac{5}{6} \approx 0.83$
Term-2, $|z| > \frac{1}{4} \approx 0.25$

So, The intersection for is, $|z| > 5/6$.

3.a] Obtain the direct form I & II and cascade form realization for the following system.

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

Here,

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

Taking Z-transform on both side, we get,

$$Y(z) = -0.1z^{-1}Y(z) + 0.2z^{-2}Y(z) + 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

$$Y(z) \{1 + 0.1z^{-1} - 0.2z^{-2}\} = X(z) \{3 + 3.6z^{-1} + 0.6z^{-2}\}$$

$$\frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$\therefore H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}} \quad \text{--- (i)}$$

Consider with general expression of DSP filter,

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad \text{--- (ii)}$$

Now, Comparing eqn (i) and (ii) we get,

$$\begin{aligned} b_0 &= 3, & b_1 &= 3.6, & b_2 &= 0.6 \\ a_1 &= 0.1, & a_2 &= -0.2 \end{aligned}$$

Then,

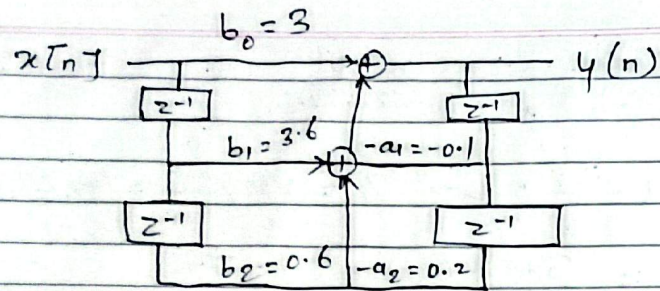


fig: Direct form I realization.

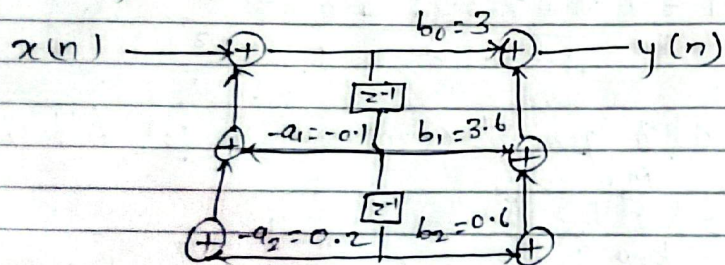


fig: Direct form II realization.

3.6] Determine the lattice coefficient corresponding to the FIR system with the system function $H(z) = 1 + \frac{7}{9}z^{-1} + \frac{3}{5}z^{-2}$

Here.

$$h(z) = 1 + \frac{7}{9}z^{-1} + \frac{3}{5}z^{-2} \quad \text{--- (i)}$$

Here. $N=2$
Now, for FIR filter,

$$H(z) = \sum_{n=0}^N a_n z^{-n}$$

$$= \sum_{n=0}^2 a_n z^{-n}$$

$$= a_0 + a_1 z^{-1} + a_2 z^{-2} \quad \text{--- (ii)}$$

from eqⁿ (i) and (ii), we get,

$$a_0 = 1, \quad a_1 = \frac{7}{9}, \quad a_2 = \frac{3}{5}$$

for Direct-to lattice Coefficient Conversion,

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m) \cdot a_m(m-k)}{1 - a_m^2(m)}$$

Here. $N=2, m=2, \text{ i.e. } M=N, N-1, N-2, \dots$

if $m=2, k=1, \text{ i.e. } k=M-1, M-2, \dots$

$$\therefore a_{2-1}(k) = \frac{a_2(k) - a_2(2) \cdot a_2(2-k)}{1 - a_2^2(2)}$$

$$a_1(k) = \frac{7/9 - \frac{3}{5} \cdot 7/9}{1 - \left(\frac{3}{5}\right)^2} = \frac{63}{80}$$

$$k_1 = 63/80$$

$$k_2 = 3/5$$

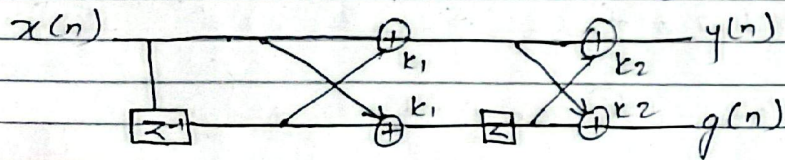


fig: Two-stage lattice structure.

4.a] The transfer function of analog filter is $H_a(s) = \frac{2}{(s+2)(s+4)}$ with $T = 0.1$ sec. Design

the IIR digital filter by using impulse invariance method.

⇒ Given.

$$H_a(s) = \frac{2}{(s+2)(s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+4)}$$

$$\frac{2}{(s+2)(s+4)} = \frac{A(s+4) + B(s+2)}{(s+2)(s+4)}$$

$$2 = A(s+4) + B(s+2)$$

When, $s = -2$,

$$2 = 2A$$

$$\therefore A = 2/2 = 1$$

Again, When, $s = -4$,

$$2 = -2B$$

$$B = \frac{2}{-2} = -1$$

So, $A = 1$, $B = -1$

$$\therefore h(s) = \frac{1}{(s+2)} - \frac{1}{(s+4)}$$

We know by IIM,

$\frac{1}{s+P_1} \xrightarrow{\text{IIM}} \frac{1}{1 - e^{P_1 T} z^{-1}}$ is the mapping formula for the impulse invariance method.

$T = 0.1$ sec,

$$P_1 = 2 \Rightarrow e^{-P_1 T} = e^{-2 \times 0.1} = e^{-0.2} = 0.8187$$

$$P_2 = 4 \Rightarrow e^{-P_2 T} = e^{-4 \times 0.1} = e^{-0.4} = 0.6703$$

$$\begin{aligned}
 H(z) &= \frac{1}{1 - e^{-0.2} z^{-1}} - \frac{1}{1 - e^{-0.4} z^{-1}} \\
 &= \frac{(1 - e^{-0.4} z^{-1}) - (1 - e^{-0.2} z^{-1})}{(1 - e^{-0.2} z^{-1})(1 - e^{-0.4} z^{-1})} \\
 &= \frac{-e^{-0.4} z^{-1} + e^{-0.2} z^{-1}}{(1 - 1.489 z^{-1} + 0.5488 z^{-2})} \\
 &= \frac{0.01484 z^{-1}}{1 - 1.489 z^{-1} + 0.5488 z^{-2}}
 \end{aligned}$$

∴

4.6] Design a digital lowpass Butterworth filter to meet the following specifications using Bilinear transformation method.

$$|H(e^{j\omega})| = \begin{cases} 0.8 & |\omega| < 0.25\pi \\ 0.25 & 0.45\pi < \omega < \pi \end{cases}$$

⇒ From given specification,

$$\begin{aligned}
 \text{Passband edge frequency } (\omega_p) &= 0.25\pi \\
 \text{passband attenuation } (A_p) &= 0.8 \\
 \text{stopband edge frequency } (\omega_s) &= 0.45\pi \\
 \text{stopband attenuation } (A_s) &= 0.25
 \end{aligned}$$

Bilinear Transformation Method (BLT), we have,

$$\text{Analog freq. } (s_2) = \frac{2 \tan(\omega/2)}{T}$$

$$\therefore s_{2p} = \frac{2 \tan(\omega_p)}{T} \quad (T=1 \text{ assume})$$

$$= 2 \tan\left(\frac{0.25\pi}{2}\right)$$

$$= 0.8284$$

Similarly,

$$s_{2s} = \frac{2 \tan(\omega_s)}{T} = 2 \tan\left(\frac{0.45\pi}{2}\right)$$

$$= 1.7081$$

Step-2] filter order (N)

$$\therefore N = \frac{\log\left[\frac{\frac{1}{A_p^2} - 1}{\frac{1}{A_s^2} - 1}\right]}{2 \log\left(\frac{s_{2s}}{s_{2p}}\right)} = \frac{\log\left[\frac{\frac{1}{(0.8)^2} - 1}{\frac{1}{(0.25)^2} - 1}\right]}{2 \log\left(\frac{1.7081}{0.8284}\right)}$$

$$= 2.26$$

$$= 3$$

Step-3] cut off frequency (ω_c)

$$\therefore \omega_c = \frac{\omega_p}{\left[\frac{1}{A_p^2} - 1 \right]^{\frac{1}{2N}}} = \frac{0.8784}{\left[\frac{1}{(0.8)^2} - 1 \right]^{\frac{1}{6}}} = 0.911$$

Step-4] Poles for $N=3$

$$\therefore P_i = + \omega_c e^{j(N+2i+1)\pi/2N}; i=0,1,2$$

When $i=0$.

$$P_0 = + \omega_c e^{j(3+0+1)\pi/6} = + \omega_c e^{j2\pi/3}$$

$$= + \omega_c \left[\cos 2\pi/3 + j \sin 2\pi/3 \right]$$

$$= + \omega_c [-0.5 + j0.86]$$

i.e poles must lie in -ve s-plane.

When $i=1$

$$P_1 = + \omega_c e^{j(3+2+1)\pi/6}$$

$$= + \omega_c e^{j\pi}$$

$$= + \omega_c$$

$$P_1 = -\omega_c / \omega_c$$

$$\therefore P_1 = -\omega_c$$

Since, $N=3$, it must have 3-poles P_1, P_2 and P_3 . Since P_1 has only real part P_2 must be complex conjugate of P_0 i.e $P_2 = P_0^*$.

$$P_2 = (-0.5 - j0.86)\omega_c$$

Step-5] The analog transfer function is.

$$H_a(s) = \frac{\omega_c^N}{(s-P_0)(s-P_1)(s-P_2)}$$

$$= \frac{\omega_c^3}{(s-P_0)(s-P_1)(s-P_2)}$$

$$= \frac{\omega_c^3}{[s - (0.5 + j0.86)\omega_c][s - (-\omega_c)][s - (0.5 - j0.86)\omega_c]}$$

$$= \frac{\omega_c^3}{(s + 0.5\omega_c - j0.86\omega_c)(s + \omega_c)(s + 0.5\omega_c + j0.86\omega_c)}$$

$$= \frac{\omega_c^3}{s^3 + 1.9\omega_c^2 s + 0.9\omega_c^2 - (0.86\omega_c)^2}$$

$$= \frac{\omega_c^3}{s^3 + 1.9\omega_c^2 s + 0.9\omega_c^2 - 0.7396\omega_c^2}$$

$$= \frac{s_c^3}{\{s^2 + s_c s + 0.25 s_c^2\} + (0.86 s_c)^2 \{s + s_c\}}$$

$$= \frac{(0.911)^3}{s^2 + 0.911s + 0.25 \times 0.911 + 0.86 \times (0.911)^2 \{s + 0.911\}}$$

$$= \frac{0.756}{(s^2 + 0.911s + 0.827)(s + 0.911)}$$

Step-6] finally the digital transfer function is,

$$H(z) = H(s) \Big|_{s = \frac{2}{T} \left(\frac{1-z^{-1}}{1+z^{-1}} \right)}$$

$$= \frac{0.756}{\left\{ \left(\frac{2}{T} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) \right)^2 + 0.911 \left(\frac{2}{T} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) \right) + 0.827 \right\} \left(\frac{2}{T} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) + 0.911 \right)}$$

⌘

5.a] Design a linear FIR filter using Kaiser window to meet the following specification
 $0.99 \leq |H(e^{j\omega})| \leq 1.01$ for $0 \leq \omega \leq 0.19\pi$
 $|H(e^{j\omega})| \leq 0.01$ for $0.21\pi \leq \omega \leq \pi$

⇒ Converting the given specification as,

$$1 - \delta_1 \leq |H(e^{j\omega})| \leq 1 + \delta_1 \text{ ; for } 0 \leq \omega \leq \omega_p$$

$$0 \leq |H(e^{j\omega})| \leq \delta_2 \text{ ; for } \omega_s \leq \omega \leq \pi$$

Then,

$$1 - 0.01 \leq |H(e^{j\omega})| \leq 1 + 0.01 \text{ ; for } 0 \leq \omega \leq 0.19\pi$$

$$0 \leq |H(e^{j\omega})| \leq 0.01 \text{ ; for } 0.21\pi \leq \omega \leq \pi$$

Then we get,

$$\delta_1 = 0.01 = \delta_2$$

$$\omega_p = 0.19\pi$$

$$\omega_s = 0.21\pi$$

for Kaiser,

① Attenuation,

$$(A) = -20 \log_{10}(\delta)$$

$$\text{where } \delta_{\min} = (\delta_1, \delta_2)$$

$$= 0.01$$

$$= -20 \log_{10}(0.01)$$

∴ Attenuation = 40 dB

2) for $A = 40$ dB, It uses the range of $21 \leq A \leq 50$

$$B = 0.5842 (A - 21)^{0.4} + 0.07886 (A - 21) \\ = 0.5842 (40 - 21)^{0.4} + 0.07886 (40 - 21) \\ = 3.39524$$

$$3) M = \frac{A \cdot B}{2.285 (\omega_s - \omega_p)} \\ = \frac{40 \cdot 3.39524}{2.285 (0.21 - 0.19)\pi} = 222.8863 \approx 223$$

$$4) \alpha = \frac{M}{2} = \frac{222.8863}{2} = 111.443 \approx 111.5$$

Now.

Kaiser window is,

$$w_x(n) = \frac{J_0 \left[\xi B \left(1 - \left(\frac{n - \alpha}{\alpha} \right)^2 \right)^{1/2} \right]}{J_0(B)} \\ = \frac{J_0 \left[3.3952 \left(1 - \left(\frac{n - 111.5}{111.5} \right)^2 \right)^{1/2} \right]}{J_0(3.3952)}$$

The desired impulse response is,

$$h_d(n) = \frac{\sin \omega_c (n - \tau)}{\pi (n - \tau)} \\ = \frac{\sin (n - \tau)}{\pi (n - \tau)}$$

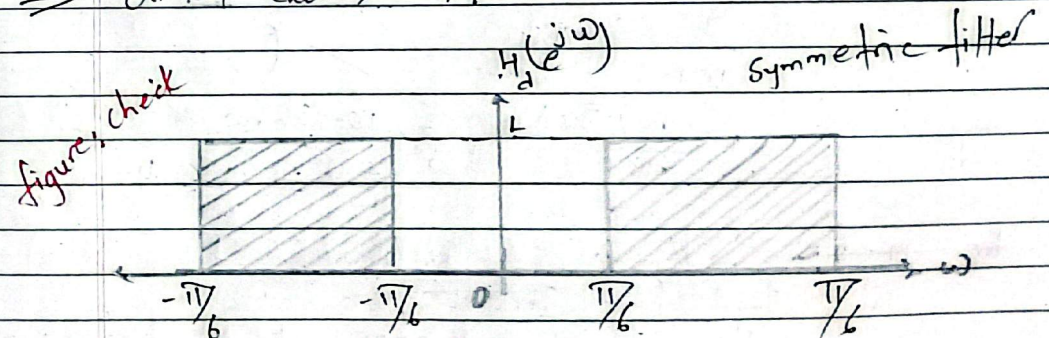
$$\tau = \frac{m-1}{2}$$

$$\therefore \tau = \frac{M}{2} = \frac{223}{2} = 111.5$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega}^{\omega} H_d(e^{j\omega}) \cdot e^{j\omega\tau} d\omega$$

5.6] Design a filter with $H_d(e^{j\omega}) = \begin{cases} 1 & -\pi/6 \leq \omega \leq \pi/6 \\ 0 & \text{otherwise} \end{cases}$ using a hanning window with $M=9$.

⇒ Given data, $M=9$



The desired impulse response sequence $h_d(n)$ is

$$\therefore h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/6}^{\pi/6} (1) \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{2\pi jn} \left[e^{j\pi/6 n} - e^{-j\pi/6 n} \right]$$

$$= \frac{1}{n\pi} \left[\frac{e^{j\pi/6 n} - e^{-j\pi/6 n}}{2j} \right]$$

$$= \frac{1}{n\pi} \sin \pi/6 n$$

$$h_d(n) = \frac{\sin(n\pi/6)}{n\pi} \quad \text{We know } \alpha = \frac{M-1}{2}$$

$$-4 \leq n \leq 4$$

$$h_d[0] = \frac{\sin(0\pi/6)}{0\pi}$$

for, $n=0$. By L-Hospital rule,

$$h_d[0] = \frac{\sin(n\pi/6)}{(n\pi)} \times \frac{1}{6}$$

$$= 1 \times \frac{1}{6} \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} \right]$$

$$= \frac{1}{6}$$

for, $n=1$,

$$h_d(1) = \frac{\sin \pi/6}{\pi} = 0.0029$$

for, $n=2$,

$$h_d(2) = \frac{\sin \pi/3}{\pi} = 0.0058$$

for, $n=3$,

$$h_d(3) = \frac{\sin(\pi/2)}{\pi} = 0.0087$$

for, $n=4$,

$$h_d(4) = \frac{\sin(2\pi/3)}{\pi} = 0.0116$$

$$\text{So, } h_d(0) = 1/6 = 0.1667$$

$$h_d(1) = 0.0029$$

$$h_d(2) = 0.0058$$

$$h_d(3) = 0.0087$$

$$h_d(4) = 0.0116$$

2) Hanning window sequence for $M=9$.

$$W_{Hn}(n) = 0.5 \left(1 - \cos \frac{2\pi n}{M-1} \right) \text{ for } n=0, 1, \dots, M-1$$

$$= 0.5 \left[1 - \frac{\cos 2\pi n}{8} \right], \quad -4 \leq n \leq 4$$

$$= 0.5 \left(1 - \cos \left(\frac{\pi n}{4} \right) \right), \quad -4 \leq n \leq 4 \quad (ii)$$

$$\text{for } n=0, W_{Hn}(0) = 0.5 (1 - \cos(0)) = 0$$

$$\text{for } n=1, W_{Hn}(1) = 0.5 (1 - \cos(\pi/4)) = 0.1464$$

$$\text{for } n=2, W_{Hn}(2) = 0.5 (1 - \cos(\pi/2)) = 0.5$$

$$\text{for } n=3, W_{Hn}(3) = 0.5 (1 - \cos(3\pi/4)) = 0.8536$$

$$\text{for } n=4, W_{Hn}(4) = 0.5 (1 - \cos(\pi)) = 1$$

3) Multiply $h_d(n)$ with W_{Hn} to get finite impulse sequence.

we know, $h(n) = h_d(n) \cdot W_{Hn}(n)$. (iii)

So,

$$h(0) = h_d(0) \cdot W_{Hn}(0) = 0$$

$$h(1) = h_d(1) \cdot W_{Hn}(1) = 0.0004$$

$$h(2) = 0.0029$$

$$h(3) = 0.0074$$

$$h(4) = 0.0116$$

$$X(k) = \sum_{n=0}^{N-1} x_1(n) \cdot e^{-j2\pi kn/N}$$

6.a] Mathematically, perform the Circular Convolution of the following two sequences $x_1[n] = \{1, 2, 3, 4\}$
 $x_2[n] = \{1, 3, 5, 7\}$

⇒ Given $N = 4$
 By Circular Convolution.

$$\begin{aligned} x_3[n] &= x_1[n] \otimes x_2[n] \\ &= \sum_{m=0}^{N-1} x_1(m) \cdot x_2(n-m) \\ &= x_1(0) \cdot x_2(n) + x_1(1) \cdot x_2(n-1) + x_1(2) \cdot x_2(n-2) \\ &\quad + x_1(3) \cdot x_2(n-3) \end{aligned}$$

$$x_3(n) = x_2(n) + 2x_2(n-1) + 3x_2(n-2) + 4x_2(n-3)$$

Where, $n = 0, 1, 2, 3$ i.e. 0 to $N-1$
 when

$n = 0$,

$$\begin{aligned} x_3(0) &= x_2(0) + 2x_2(-1) + 3x_2(-2) + 4x_2(-3) \\ &= x_2(0) + 2x_2(4-1) + 3x_2(4-2) + 4x_2(4-3) \\ &= x_2(0) + 2x_2(3) + 3x_2(2) + 4x_2(1) \\ &= 1 + 2 \times 7 + 3 \times 5 + 4 \times 3 \\ &= 42 \end{aligned}$$

$$x_3(n) = X_3(k) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2(n-m) \cdot N$$

When $n = 1$,

$$\begin{aligned} x_3(1) &= x_2(1) + 2x_2(0) + 3x_2(-1) + 4x_2(-2) \\ &= x_2(1) + 2x_2(0) + 3x_2(3) + 4x_2(2) \\ &= 3 + 2 \times 1 + 3 \times 7 + 4 \times 5 \\ &= 46 \end{aligned}$$

When $n = 2$,

$$\begin{aligned} x_3(2) &= x_2(2) + 2x_2(1) + 3x_2(0) + 4x_2(3) \\ &= 5 + 2 \times 3 + 3 \times 1 + 4 \times 7 \\ &= 42 \end{aligned}$$

When $n = 3$,

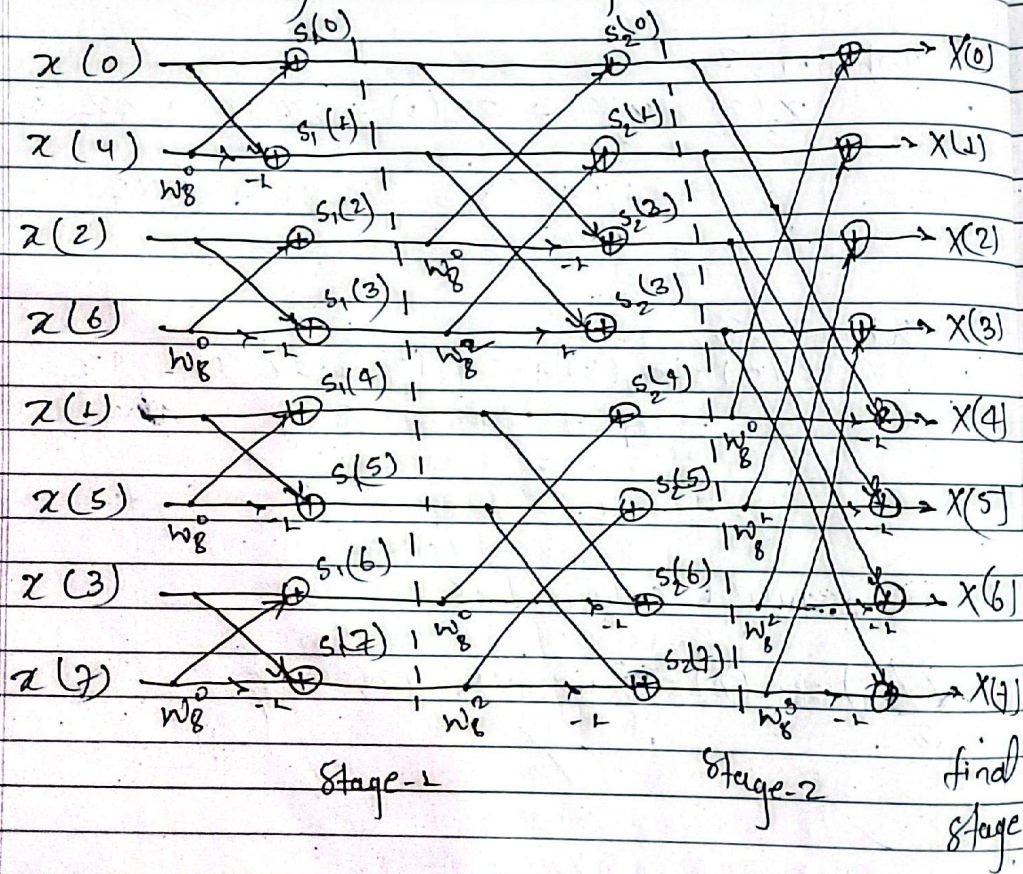
$$\begin{aligned} x_3(3) &= x_2(3) + 2x_2(2) + 3x_2(1) + 4x_2(0) \\ &= 7 + 2 \times 5 + 3 \times 3 + 4 \times 1 \\ &= 30 \end{aligned}$$

So, required solution is,

$$X_3(k) = x_3(n) = \{42, 46, 42, 30\}$$

6.b] Use the 8-point radix-2 DST-FFT algorithm to find the DFT of the sequence.
 $x(n) = \{0.707, 1, 0.707, 0, -0.707, -1, -0.707, 0\}$

⇒ We know by DST-FFT Algorithm:



Where, $w_8^0 = 1$
 $w_8 = e^{-j2\pi/8} = e^{-j\pi/4}$
 $= \cos \pi/4 - j \sin \pi/4$
 $= +0.707 - j0.707$

$w_8^1 = 0.707 - j0.707$

$w_8^2 = e^{-j\pi/2} = \cos \pi/2 - j \sin \pi/2$
 $= -j$

$w_8^3 = w_8^1 \cdot w_8^2 = (-j)(0.707 - j0.707)$
 $= -j0.707 - 0.707$
 $= -0.707 - j0.707$

Output of Stage-1:

$S_1(0) = x(0) + x(4) \cdot w_8^0 = 0.707 - 0.707 = 0$
 $S_2(1) = x(1) - x(5) w_8^0 = 0.707 + 0.707 = 1.414$
 $S_1(2) = x(2) + x(6) w_8^0 = 0$
 $S_1(3) = x(2) - x(6) w_8^0 = 1.414$
 $S_1(4) = x(4) + x(0) w_8^0 = 0$
 $S_1(5) = x(4) - x(0) w_8^0 = 2$
 $S_1(6) = x(6) + x(2) w_8^0 = 0$
 $S_1(7) = x(6) - x(2) w_8^0 = 0$

Output of stage-2:

$$\begin{aligned} s_2(0) &= s_1(0) + s_1(2) = 0 \\ s_2(1) &= s_1(1) + s_1(3) = 1.414 - j1.414 \\ s_2(2) &= [s_1(0) - s_1(2)] w_8^0 = 0 \\ s_2(3) &= [s_1(1) - s_1(3)] w_8^2 = 1.414 + j1.414 \\ s_2(4) &= s_1(4) + s_1(6) = 0 \\ s_2(5) &= s_1(5) + s_1(7) = 2 \\ s_2(6) &= [s_1(4) - s_1(6)] w_8^0 = 0 \\ s_2(7) &= [s_1(5) - s_1(7)] w_8^2 = 2 \end{aligned}$$

Output of final stage:

$$\begin{aligned} X(0) &= s_2(0) + s_2(4) = 0 \\ X(1) &= s_2(1) + s_2(5) = 2.828 - j2.828 \\ X(2) &= s_2(2) + s_2(3) = 0 \\ X(3) &= s_2(6) + s_2(7) = 0 \\ X(4) &= [s_2(0) - s_2(4)] w_8^1 = 0 \\ X(5) &= [s_2(1) - s_2(5)] w_8^2 = 0 \\ X(6) &= [s_2(2) - s_2(3)] w_8^3 = 0 \\ X(7) &= [s_2(6) - s_2(7)] w_8^3 = 2.828 + j2.828 \end{aligned}$$

So, Required solution.

$$X(n) = \{0, 2.828 - j2.828, 0, 0, 0, 0, 2.828 + j2.828\}$$

Q.N. 7 [a]. LTI system & Convolution summation of discrete system.

⇒ An LTI (Linear time invariant) system is a system that satisfies:

i) Linearity:

$$\begin{aligned} \text{If } x_1[n] \rightarrow y_1[n] \text{ \& } x_2[n] \rightarrow y_2[n] \text{ then} \\ x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n] \end{aligned}$$

ii) Scaling, $ax_2[n] \rightarrow ay_2[n]$

ii) Time invariance:

$$\text{If input } x[n] \text{ produces output } y[n], \text{ then} \\ x[n - n_0] \rightarrow y[n - n_0]$$

#. Convolution summation of discrete time system.

for a discrete time LTI system, the output is obtained by convolution of input $x[n]$ with impulse response $h[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

→ properties of Convolution

- Commutative
- Associative
- Distributive

7.C] Frequency response of LTI system.

→ The frequency response of a discrete time LTI system describes how the system modifies the magnitude and phase of different frequency components of the input signal.

→ It is defined as the Fourier transform of the impulse response.

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-j\omega n}.$$

→ Relation with z-transform,

If the $H(z)$ is the z-transform of $h(n)$, then the frequency response is obtained by evaluating $H(z)$ on the unit circle;

$$H(z) = H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}}$$

→ Magnitude and phase response,

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j\angle H(e^{j\omega})}$$

$|H(e^{j\omega})| \rightarrow$ magnitude response.

$\angle H(e^{j\omega}) \rightarrow$ phase response.

Importance :-

→ used to classify low pass, high pass, band pass, band stop.

→ Determines stability & distortion

→ filter design and signal analysis

basic elements DSP:

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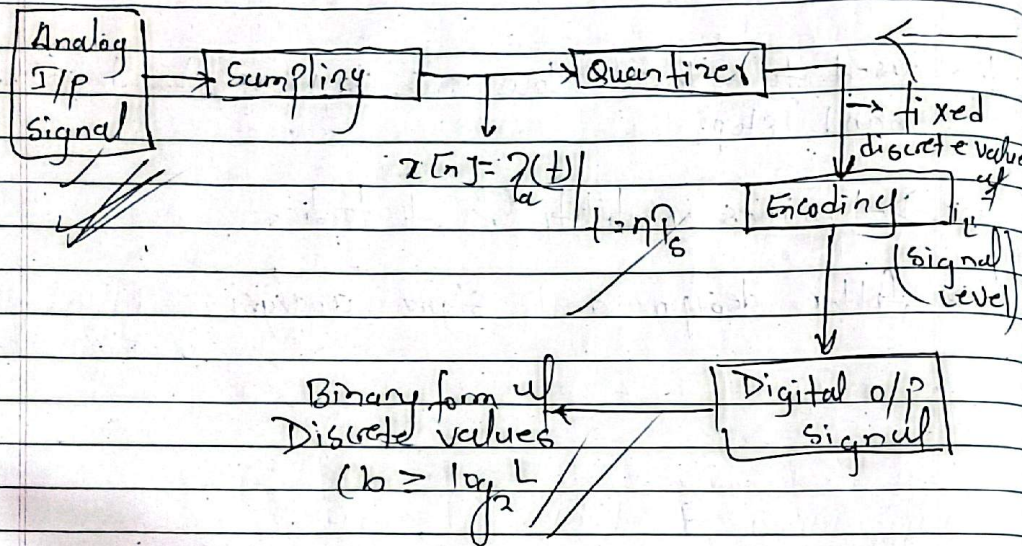


fig: Block diagram of Digital signal processing.

set-2

chance Exam.

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1.A] Write the advantages of digital signal over analog signal. Define Energy and power signals.

⇒ The advantages of digital signal over analog signal:-

- i) Noise Immunity:
Digital signals are less affected by noise and interference compared to analog signals.
- ii) Error detection and correction.
Error introduced during transmission can be detected and corrected in digital system.
- iii) Easy storage and processing:
Digital signals can be easily stored, processed and manipulated using computer and digital hardware.
- iv) High reliability:
Digital systems maintain signal quality over long distance without degradation.
- v) flexibility:
Digital signals can be compressed encrypted and multiplexed efficiently.

vii) Reproducibility :-

Digital signals can be copied multiple times without loss of quality.

viii) Compatibility with digital systems

Digital signals are suitable for modern communication and signal processing systems.

#. Energy Signal :-

A signal is called an energy signal, if it has finite energy and zero average power.

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \quad \text{for discrete time}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{for continuous time}$$

→ Energy signals are non-periodic in nature.

→ Energy signals are bandlimited signals.

#. Power Signal :-

A signal is called power signal, if it has finite power and infinite energy.

$$P_{ave} = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

where, $N \rightarrow$ fundamental period.

$$P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$\therefore 0 < P < \infty$$

→ Power signals are periodic in nature.

→ Power signals are band unlimited signals.

2.6] Determine the Convolution sum of two sequences using graphical method where bold letter denotes origin.

$$x(n) = \{ \mathbf{4}, 2, 1, 3 \}, h(n) = \{ 1, \mathbf{2}, 2, 1 \}$$

⇒ Here.

$$x(n) = \{ 4, 2, 1, 3 \}$$

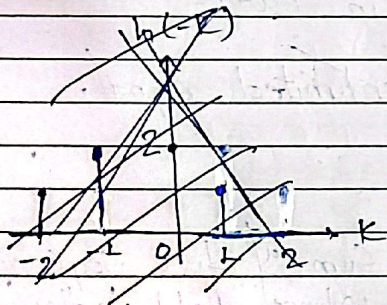
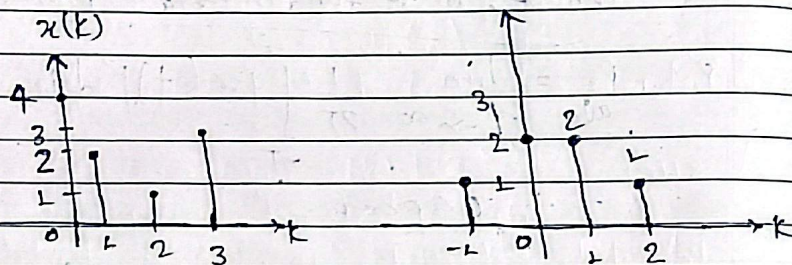
$$h(n) = \{1, 2, 2, 1\}$$

The o/p. sequence starts at $n = 0 \Rightarrow H = -1$
 we know, $\therefore$ length of output $y(n) = 4 + 4 - 1 = 7$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) \quad \text{--- (i)}$$

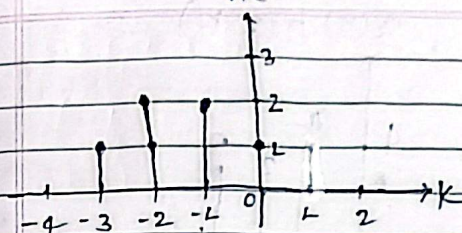
By graphical method, $n \Rightarrow (-1 \text{ to } 5)$

1)

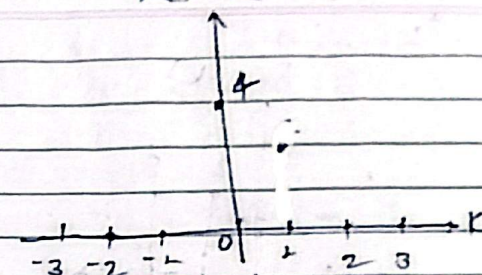


When, $n = -1$, so, $h(-1-k)$

$h(-1-k)$



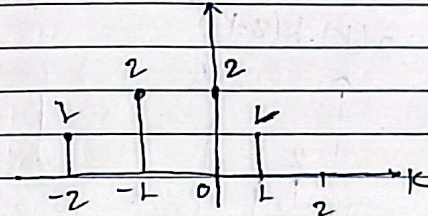
$x(k) \cdot h(-1-k)$



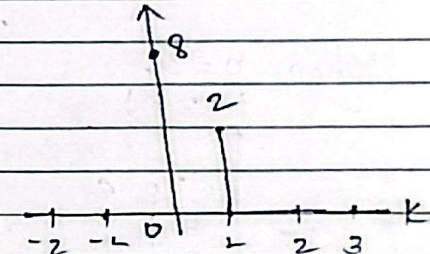
$$\therefore y(-1) = 4$$

When, $n = 0$,

$h(-k)$



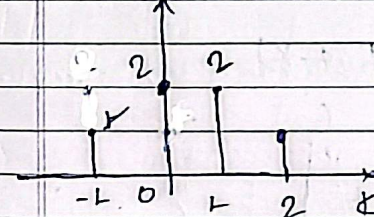
$x(k) \cdot h(-k)$



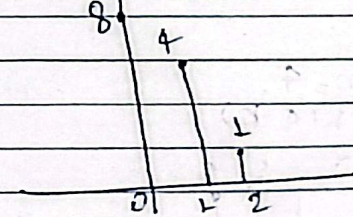
$$\therefore y(0) = 4$$

When, $n = 1$,

$h(1-k)$

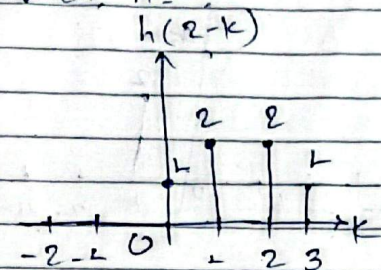


$x(k) \cdot h(1-k)$

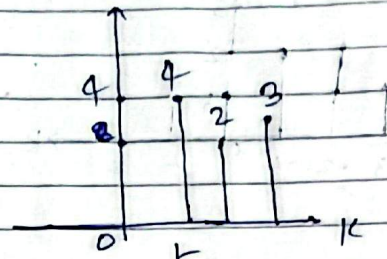


$$\therefore y(1) = 12$$

When $n=2$



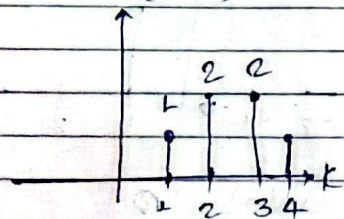
$x(k) \cdot h(2-k)$



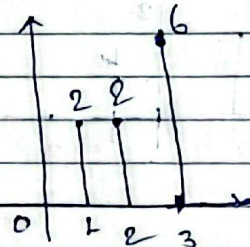
$$\therefore y(2) = 13$$

When $n=3$:

$h(3-k)$



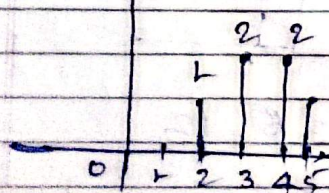
$x(k) \cdot h(3-k)$



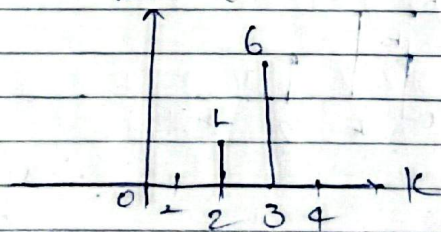
$$\therefore y(3) = 10$$

When $n=4$.

$h(4-k)$



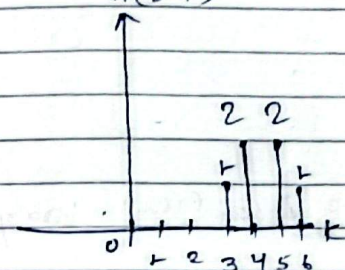
$x(k) \cdot h(4-k)$



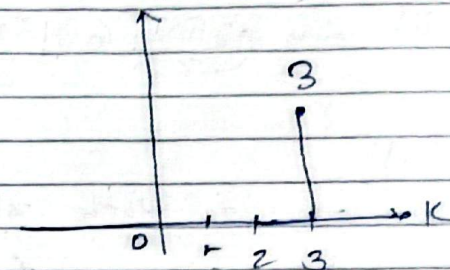
$$\therefore y(4) = 7$$

When $n=5$,

$h(5-k)$



$x(k) \cdot h(5-k)$



$$\therefore y(5) = 3$$

So,

the required solution is.

$$y(n) = \{ 4, 10, 13, 13, 10, 7, 3 \}$$

2.a) Determine which signals are periodic and compute fundamental period.

i) $\cos\left(\frac{\pi n^2}{8}\right)$

ii) $\sin\left(\frac{3\pi n}{2}\right) \cdot \cos\left(\frac{5\pi n}{4}\right)$

⇒ i) Comparing with $x[n] = A \cos(\omega_m)$. we get,

$$\omega_m = \frac{\pi n}{8}$$

$$2\pi f = \frac{\pi n}{8} \quad [\because \omega_0 = 2\pi f]$$

$$f = \frac{\pi}{16} \quad \text{where } n = 0, 1, 2, \dots$$

$$f = \frac{k}{N} = \frac{n}{16}$$

given signal are periodic because the frequency is the rational so,

fundamental period $(N) = 16$ #

ii) Comparing $x(n) = A \sin(\omega_1 n) \cdot \cos(\omega_2 n)$

for, $\sin(\omega_1 n)$,

$$\omega_1 n = \frac{3\pi n}{2}$$

$$2\pi f_1 = \frac{3\pi n}{2}$$

$$f_1 = \frac{3}{4} = \frac{k}{N_1}, \quad N_1 = 4$$

Again, $\cos(\omega_2 n)$,

$$\omega_2 n = \frac{5\pi n}{4}$$

$$2\pi f_2 = \frac{5\pi n}{4}$$

$$f_2 = \frac{5}{8} = \frac{k}{N_2}, \quad N_2 = 8$$

So,

~~$$f = f_1 + f_2 = \frac{3}{4} + \frac{5}{8} = \frac{11}{8}$$~~

given signal are periodic, since the frequency are rational, now,

fundamental period $(N) = 4 \times 2 = 8$ #

2.6] Define Z-transform and Region of Convergence.
Find Z-transform of the following signals.

$$x(n) = a^n u(n)$$

⇒ Consider discrete sequence $x(n)$ whose Z-transform is $x(n) \xrightarrow{Z} X(z)$.

i.e. $X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$

→ All the value of z in which $x(z)$ is finite is called Region of Convergence.

Here Given $x(n) = a^n u(n)$

$u(n)$ is unit step signal, we know,

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Z-transform of $u(n)$:-

$$Z \{ a^n u(n) \} = \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n \cdot 1 \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n \cdot z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

Using, $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad |r| < 1 \quad \text{So,}$$

$$X(z) = \frac{1}{1 - az^{-1}}$$

For, $|az^{-1}| < 1$

$$\left| \frac{a}{z} \right| < 1$$

$$\therefore |z| > |a|$$

3 a) Determine the inverse z-transform of $H(z) = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + 3.5z^{-1} + 1.5z^{-2}}$ for possible for.

$\Rightarrow$ Here Given.

$$H(z) = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + 3.5z^{-1} + 1.5z^{-2}}$$

Converting into positive power of z .

$$H(z) = \frac{1 + \frac{3}{z} + \frac{2}{z^2}}{1 + \frac{3.5}{z} + \frac{1.5}{z^2}}$$

$$H(z) = \frac{z^2 + 3z + 2}{z^2 + 3.5z + 1.5}$$

~~$$H(z) = \frac{(z+3)(z+2)}{z(z+1)}$$~~

numerator. $z^2 + 3z + 2 = (z+2)(z+1)$

denominator. $z^2 + 3.5z + 1.5 = (z+3)(z+0.5)$

So,

$$H(z) = \frac{(z+1)(z+2)}{(z+3)(z+0.5)}$$

By partial fraction,

$$\frac{(z+1)(z+2)}{(z+3)(z+0.5)} = \frac{A}{z+3} + \frac{B}{z+0.5}$$

$$(z+1)(z+2) = A(z+0.5) + B(z+3)$$

At $z = -3$, $(-2) \times (-1) = -2.5A$

$$2 = -2.5A$$

$$\therefore A = \frac{2}{-2.5} = -0.8$$

Again, At $z = -0.5$,

$$(0.5) \times (1.5) = 2.5B$$

$$0.75 = 2.5B$$

$$\therefore B = \frac{0.75}{2.5} = 0.3$$

So,

$$H(z) = 1 + \frac{(-0.8)}{z+3} + \frac{0.3}{(z+0.5)}$$

$$= 1 + \frac{0.3}{(z+0.5)} + \frac{0.8}{(z+3)}$$

Taking ~~z~~ z^{-1} form,

$$H(z) = 1 + \frac{0.3z^{-1}}{(1+0.5z^{-1})} - \frac{0.8z^{-1}}{(1+3z^{-1})}$$

We know,

$$z^{-1} \left(\frac{z^{-1}}{(1+az^{-1})} \right) = (-a)^{n-1} u[n-1]$$

$$\text{So, } h(n) = \delta(n) + 0.3(-0.5)^{n-1} u[n-1] - 0.8(-3)^{n-1} u[n-1]$$

for, for, causal system,

$$\text{for: } |z| > 3$$

Other possible for, Poles at $z = -3, z = -0.5$

1. $|z| > 3$ for causal
2. $|z| < 0.5$ for anticausal
3. $0.5 < |z| < 3$ for two sided.

3.6] Obtain the direct form I & direct II realization for the system described by the following equation

$$y(n) = 2x(n) + 0.3x(n-1) + 0.5x(n-2) - 0.7y(n-1) - 0.9y(n-2)$$

$\Rightarrow$ here
Given,

$$y(n) = 2x(n) + 0.3x(n-1) + 0.5x(n-2) - 0.7y(n-1) - 0.9y(n-2)$$

Taking z-transform on both sides,

$$Y(z) = 2X(z) + 0.3z^{-1}X(z) + 0.5z^{-2}X(z) - 0.7z^{-1}Y(z) - 0.9z^{-2}Y(z)$$

$$Y(z) + 0.7z^{-1}Y(z) + 0.9z^{-2}Y(z) = 2X(z) + 0.3z^{-1}X(z) + 0.5z^{-2}X(z)$$

$$(1 + 0.7z^{-1} + 0.9z^{-2})Y(z) = (2 + 0.3z^{-1} + 0.5z^{-2})X(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{2 + 0.3z^{-1} + 0.5z^{-2}}{1 + 0.7z^{-1} + 0.9z^{-2}} \quad \text{--- (1)}$$

The general eqn of IIR filter,

$$h(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (i)$$

Comparing eqⁿ (i) and (ii) we get,

$$b_0 = 2, \quad b_1 = 0.3, \quad b_2 = 0.5$$

$$a_1 = 0.7, \quad a_2 = 0.9$$

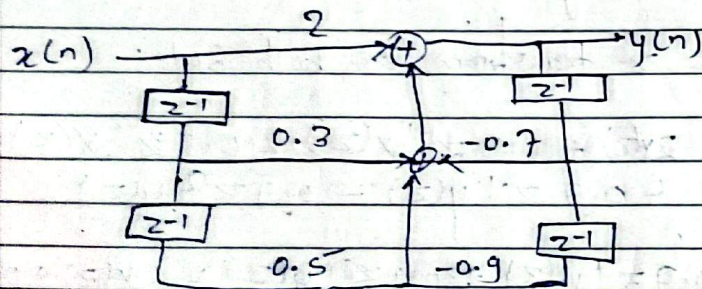


fig: Direct form -I realization.

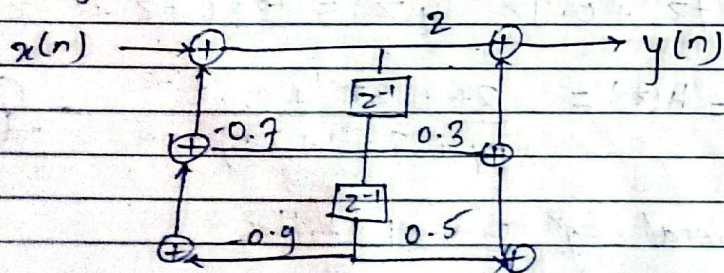


fig: Direct form -II realization.

4. a) Obtain the lattice ladder structure of the discrete time system described by the difference eqⁿs.

$$y(n) = \frac{-3}{4} y(n-1) + \frac{1}{4} y(n-2) + x(n) + \frac{1}{2} x(n-1)$$

⇒ Here

$$y[n] = \frac{-3}{4} y[n-1] + \frac{1}{4} y[n-2] + x[n] + \frac{1}{2} x[n-1]$$

Taking z-transform, we get.

$$Y(z) = \frac{-3}{4} z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) + X(z) + \frac{1}{2} z^{-1} X(z)$$

$$Y(z) + \frac{3}{4} z^{-1} Y(z) - \frac{1}{4} z^{-2} Y(z) = X(z) + \frac{1}{2} z^{-1} X(z)$$

$$\therefore \frac{Y(z)}{X(z)} = H(z) = \frac{1 + \frac{1}{2} z^{-1}}{1 + \frac{3}{4} z^{-1} - \frac{1}{4} z^{-2}}$$

$$= \frac{1 + 0.5 z^{-1}}{1 + 0.75 z^{-1} - 0.25 z^{-2}}$$

$$= \frac{1 + 0.5 z^{-1}}{1 + 0.75 z^{-1} - 0.25 z^{-2}}$$

$$\therefore H(z) = \frac{B_M(z)}{A_N(z)} = \frac{1 + 0.5 z^{-1}}{1 + 0.75 z^{-1} - 0.25 z^{-2}}$$

$$= \frac{\sum_{k=0}^N b_m(k) z^{-k}}{1 + \sum_{k=1}^N a_n(k) \cdot z^{-k}}$$

Here,

$$B_m(z) = 1 + 0.5z^{-1}, \quad b_0=1, b_1=0.5, \quad M=1$$

$$A_n(z) = 1 + 0.7z^{-1} + -0.25z^{-2}, \quad N=2$$

we have, $a_2(0)=1, a_2(1)=0.7, a_2(2)=-0.25$

$$k_2 = a_2(2) = -0.25$$

from, Direct to lattice ^{Coeffs} conversion

$$a_{m+1}(k) = \frac{a_m(k) - a_m(m) \cdot a_m(m-k)}{1 - a_m(m)}$$

$N=2, m=2, 1$
for, $m=2, k=1$,

$$\therefore a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(2-1)}{1 - a_2(2)}$$

$$= \frac{0.7 + 0.25 \times 0.7}{1 - (-0.25)^2}$$

$$= \frac{0.7 + 0.175}{0.75} = \frac{7}{6}$$

$$= 1.1667$$

The lattice coefficient are, for lattice structure,

$$k_1 = 7/6 = 1.1667$$

$$k_2 = -0.25$$

for, ladder structure, we have,

$$c_m = b_m - \sum_{i=m+1}^M c_i a_i(i-m), \quad m = M, M-1, \dots, 0$$

$$c_1 = b_1 = 0.5$$

$$\begin{aligned} c_0 &= b_0 - \sum_{i=0}^1 c_i a_i(i-0) = b_0 - [c_1 a_1(1)] \\ &= 1 - [0.5 \times 1.1667] \\ &= 0.4167 \end{aligned}$$

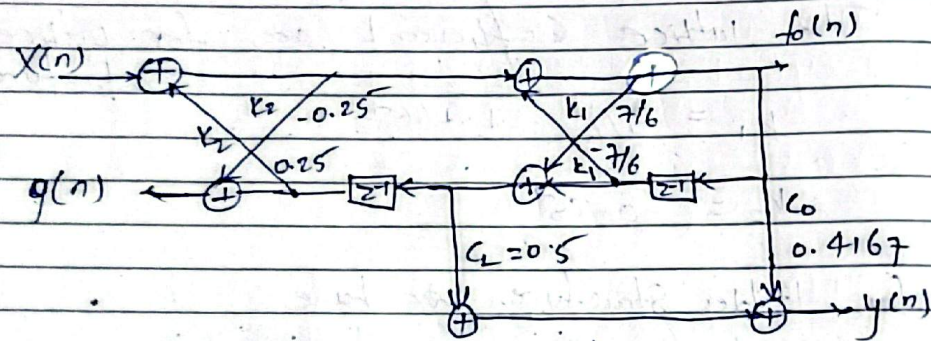
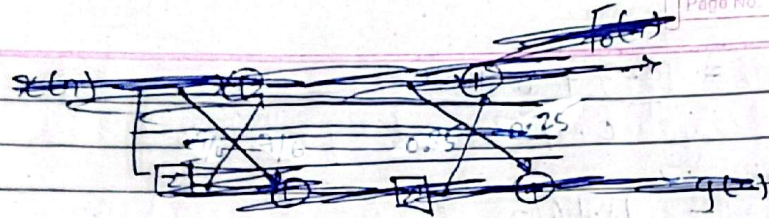


fig: lattice-ladder structure.

4.6] Why do we need DFT when we have DFT? Determine the circular convolution of the sequence $x_1(n) = \{4, 3, 2, 1\}$ & $x_2(n) = \{8, 7, 6, 5\}$. Also verify your answer.

⇒ DFT

- ↳ Defined for infinite length discrete time signals.
- ↳ Frequency is continuous.
- ↳ Can not be computed directly on a digital computer.

DFT,

- ↳ Defined for finite length sequence.
- ↳ discrete frequency.
- ↳ Computationally efficient
- ↳ Suitable for digital signal processing.

So, DFT is needed because DTFT is in practical for numerical computation, whereas DFT allows efficient digital processing using algorithms like FFT.

Here, $N = 4$

By Circular Convolution.

unit - s

$$(i) \quad \frac{1}{s - p_1} \rightarrow \frac{1}{1 - e^{p_1 T} z^{-1}}$$

$$(ii) \quad \frac{s + a}{(s + a)^2 + b^2} \rightarrow \frac{1 - e^{-aT} [\cos bT] z^{-1}}{1 - 2e^{-aT} [\cos bT] z^{-1} + e^{-2aT} z^{-2}}$$

$$(iii) \quad \frac{b}{(s + a)^2 + b^2} \rightarrow \frac{e^{-aT} [\sin bT] z^{-1}}{1 - 2e^{-aT} [\cos bT] z^{-1} + e^{-2aT} z^{-2}}$$

(1) Impulse invariance method,

Q:1] here, Given,

Sampling frequency (f_s) = 5 Hz
 Analog transfer function is,

$$h_a(s) = \frac{2}{(s+1)(s+2)}$$

$$h_a(s) = \frac{A_1}{(s+1)} + \frac{B}{(s+2)} \quad \text{--- (i)}$$

By Partial fraction,

$$\frac{2}{(s+1)(s+2)} = \frac{A_1(s+2)}{(s+1)(s+2)} + \frac{B_2(s+1)}{(s+1)(s+2)}$$

$$2 = A_1(s+2) + A_2(s+1)$$

At $s = -1$, then,

$$\therefore A_1 = 2$$

At $s = -2$ then, $A_2 = -2$

from eqn (i) becomes,

$$h_a(s) = \frac{2}{(s+1)} - \frac{2}{(s+2)}$$

To obtain z-transform using ITM,

we know,

$$\frac{1}{s - p_1} \rightarrow \frac{1}{1 - e^{p_1 T} z^{-1}} \quad \text{--- (ii)}$$

$$\text{Sampling time } (T) = \frac{1}{f_s} = \frac{1}{5} = 0.2 \text{ sec.}$$

The poles $p_1 = -1$, $p_2 = -2$

using eqn (ii), we obtain,

$$\frac{1}{s+1} = \frac{1}{1 - e^{-1 \times 0.2} z^{-1}} = \frac{1}{1 - e^{-0.2} z^{-1}}$$

$$\frac{1}{s+2} = \frac{1}{1 - e^{-2 \times 0.2} z^{-1}} = \frac{1}{1 - e^{-0.4} z^{-1}}$$

The transfer function, of digital filter is,

$$H(z) = \sum_{i=1}^N \frac{A_i}{1 - e^{p_i T} z^{-1}}$$

So,

$$H(z) = \frac{A_1}{1 - e^{p_1 T} z^{-1}} + \frac{A_2}{1 - e^{p_2 T} z^{-1}}$$

$$= \frac{2}{1 - e^{-0.2} z^{-1}} - \frac{2}{1 - e^{-0.4} z^{-1}}$$

$$= \frac{2}{1 - e^{-0.2} \frac{1}{z}} - \frac{2}{1 - e^{-0.4} \frac{1}{z}}$$

$$= \frac{2z}{z - e^{-0.2}} - \frac{2z}{z - e^{-0.4}}$$

$$= \frac{2z(z - 0.67) - 2z(z - 0.81)}{(z - 0.81)(z - 0.67)}$$

$$= \frac{2z^2 - 1.34z - 2z^2 + 1.62z}{z^2 - 0.67z - 0.81z + 0.54}$$

$$= \frac{0.28z}{z^2 - 1.48z + 0.54}$$

Which is required solution.

Q7.

Here,

analog transfer function.

$$H(s) = \frac{1}{(s+0.5)(s^2+0.5s+2)}$$

By Partial fraction,

$$H(s) = \frac{A_1}{(s+0.5)} + \frac{A_2s + A_3}{(s^2+0.5s+2)} \quad \text{--- (i)}$$

$$1 = A_1(s^2+0.5s+2) + (A_2s+A_3)(s+0.5)$$

$$1 = A_1s^2 + 0.5A_1s + 2A_1 + A_2s^2 + 0.5A_2s + A_3s + 0.5A_3$$

$$\textcircled{1} = (A_1+A_2)s^2 + (0.5A_1+0.5A_2+A_3)s + 2A_1+0.5A_3$$

Equating,

$$A_1 + A_2 = 0$$

$$\therefore A_1 = -A_2 \quad \text{--- (*)}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$0.5A_1 + 0.5A_2 + A_3 = 0$$

$$-0.5A_2 + 0.5A_2 + A_3 = 0$$

$$\therefore A_3 = 0$$

$$2A_1 + 0.5A_3 = 1$$

$$2A_1 = 1$$

$$\therefore A_1 = 1/2 = 0.5$$

$$A_2 = -1/2 = -0.5$$

Now, eqn (i) becomes,

$$H(s) = \frac{0.5}{(s+0.5)} - \frac{0.5s}{(s^2+0.5s+2)}$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5s}{(s^2+0.5s+2)}$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5s}{\left(s^2 + 2\left(\frac{1}{4}\right)s + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 + 2\right)}$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5s}{(s+\frac{1}{4})^2 + (\frac{3}{4})^2}$$

$$= \frac{0.5}{s+0.5} - \frac{0.5[s+0.25]}{(s+0.25)^2 + (1.39)^2} \quad (ii)$$

We know

$$\frac{1}{s-p} \rightarrow \frac{1}{1-e^{pT}z^{-1}}$$

and,

$$\frac{s+a}{(s+a)^2+b^2} \rightarrow \frac{1-e^{-aT}(\cos bT) z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Where, $b =$ ~~At initial, $T=1$~~

$$p_0 = 0.5, \quad a = 0.25, \quad b = 1.39$$

Now,

The transfer function of digital filter is.
eqn (ii) becomes,

$$H(z) = \frac{0.5}{1-e^{0.5T}z^{-1}} - \frac{0.5[1-e^{-0.25T} \cos(1.39T)]z^{-1}}{1-2e^{-0.25T} \cos(1.39T)z^{-1}+e^{-0.5T}z^{-2}}$$

$$s^2 + 2(\frac{1}{4})s + (\frac{1}{4})^2$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5[s+0.25-(0.25)]}{(s+0.25)^2 + (1.39)^2}$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5[s+0.25]}{(s+0.25)^2 + (1.39)^2} - \frac{0.5[-0.25]}{1}$$

$$= \frac{0.5}{(s+0.5)} - \frac{0.5[s+0.25]}{(s+0.25)^2 + (1.39)^2} + \frac{0.13}{1.39} \left(\frac{1.39}{1} \right)$$

$$1-2e^{-0.25T} \cos(1.39T)z^{-1} + e^{-0.5T}z^{-2}$$

$$+ 0.09 \left[\frac{e^{-0.25T} \sin(1.39T) \cdot z^{-1}}{1-e^{-0.25T} \cos(1.39T)z^{-1} + e^{-0.5T}z^{-2}} \right]$$

$$\Rightarrow H(z) = \frac{0.5}{1-e^{0.5T}z^{-1}} - \frac{0.5[1-e^{-0.25T} \cos(1.39T)]z^{-1}}{1-2e^{-0.25T} \cos(1.39T)z^{-1}+e^{-0.5T}z^{-2}}$$

#

Q.7. Alternate eqⁿ:-

$$h_a(s) = \frac{1}{(s+4)^2 + 16} \quad \text{--- (i)}$$

$$\omega_c = \frac{\omega}{T}$$

As we know,

$$s = \sigma + j\omega_c$$

$$\text{So, } (s^2 + 4)^2 + 16 = 0$$

$$(s^2 + 4)^2 - (4j)^2 = 0$$

$$\therefore (s + 4 - 4j)(s + 4 + 4j) = 0$$

$$s = -4 + 4j$$

So,

$$\omega_c = 4$$

$$\text{We know, } \omega_c = \frac{\omega}{T}$$

$$T = \frac{\omega}{\omega_c} = \frac{\pi/2}{4} = \frac{\pi}{8}$$

Now,

$$h_a(s) = \frac{1}{(s+4)^2 + (4)^2}$$

$$= \frac{1}{4} \left[\frac{4}{(s+4)^2 + (4)^2} \right] \quad \text{--- (i)}$$

We know,

$$\frac{b}{(s+a)^2 + b^2} \rightarrow \frac{e^{-at} \sin(bt) \cdot z^{-1}}{1 - 2e^{-at} \cos(bt) \cdot z^{-1} + e^{-2at} z^{-2}}$$

$$\text{Where, } T = \pi/8, a = 4, b = 4$$

So, The required solution is,

$$H(z) = \frac{e^{-4 \cdot \pi/8} \sin(4 \cdot \pi/8) \cdot z^{-1}}{1 - 2e^{-4 \cdot \pi/8} \cos(4 \cdot \pi/8) \cdot z^{-1} + e^{-2 \cdot 4 \cdot \pi/8} z^{-2}}$$

$$= \frac{e^{-\pi/2} \sin(\pi/2) \cdot z^{-1}}{1 - 2e^{-\pi/2} \cos(\pi/2) \cdot z^{-1} + e^{-\pi} z^{-2}}$$

$$= \frac{e^{-\pi/2} \sin(\pi/2) \cdot z^{-1}}{1 - 2e^{-\pi/2} \cos(\pi/2) \cdot z^{-1} + e^{-\pi} z^{-2}}$$

$$= \frac{e^{-\pi/2} \sin(\pi/2) \cdot z^{-1}}{1 - 2e^{-\pi/2} \cos(\pi/2) \cdot z^{-1} + e^{-\pi} z^{-2}}$$

$$= \frac{0.21 z^{-1}}{1 + 0.04 z^{-2}}$$

$$= \frac{0.21}{z} \cdot \frac{1}{1 + 0.04 \frac{1}{z^2}}$$

$$= \frac{0.21 z}{z^2 + 0.04}$$

Bilinear transformation.

$$\left[s = \frac{2}{T} \tan\left(\frac{\omega}{2}\right) \right] \text{ or } \left[\omega = \frac{2}{s} \tan\left(\frac{\omega}{2}\right) \right]$$

$$\rightarrow s = \frac{2}{T} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right) \quad \left[s = \sigma + j\omega \right]$$

Butterworth filter:

Step-1]

a) TLM $s = \frac{\omega}{T}$

b) BLT $s = \frac{2}{T} \tan\left(\frac{\omega}{2}\right)$

Where s frequency of analog filter.

ω frequency of digital filter.

T sampling time.

Step-2]

Obtain the order of filter N (filter length)

$$N = \log_{10} \left[\frac{10^{0.1 A_s} - 1}{10^{0.1 A_p} - 1} \right] \quad \text{(dB)} \rightarrow \text{decibels.}$$

or,

$$2 \log_{10} \left(\frac{s_z s}{s_p} \right)$$

~~or,~~

$$N = \log_{10} \left[\frac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1} \right] \cdot \frac{2 \log_{10} (s_z s / s_p)}{2 \log_{10} (s_z s / s_p)}$$

Step-3] obtain the cut off frequency

a) $\omega_c = \frac{\omega_c}{T}$ by πM

$\omega_c = \frac{2}{T} \tan\left(\frac{\omega_c}{2}\right)$, by BZT

~~if~~ if ω_c not given,

i) for dB, $\omega_c = \frac{\omega_p}{\left(\frac{1}{\frac{1}{2} - A_p - 1}\right)^{1/2N}}$

ii) Not dB, $\omega_c = \frac{\omega_p}{\left[10^{0.1 A_p - 1}\right]^{1/2N}}$

Step-4] determine the poles.

$P_i = \pm \omega_c e^{j(N+2i+L)\pi/2N}$
 $i = 0, 1, 2, \dots, N-L.$

Step-5] obtain the analog transformation

$h_a(s) = \frac{N}{s^N (s-p_0)(s-p_1)\dots\dots}$

Step-6] obtain the digital filter BZT

$H(z) = \left(\frac{1}{s}\right) \frac{2}{T} \left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ by πM or BZT

Where ω_c cut-off edge frequency in digital term.

#

Q.N.7.

Here, order of filter $N = 2$

Cut off frequency (f_c) = 1 MHz = 1000 kHz

Sampling frequency (f_s) = 10^4 sample/sec.
= 10,000 Hz.

We know,

$$f_c = \frac{f_s}{F_s} = \frac{10000}{10000} = 0.1 \text{ cycle/sec.}$$

Cut-off frequency in digital,

$$\omega_c = 2\pi f_c = 2\pi \times 0.1 \\ = 0.2\pi \text{ radian/sample.}$$

By Bilinear transformation,

$$s_c = \frac{2}{T} \tan \frac{\omega_c}{2} \\ = \frac{2}{1/f_c} \tan \left(\frac{0.2\pi}{2} \right)$$

$$= 2 \times 10000 \tan \left(0.2\pi/2 \right) \\ = 6498.39 \text{ radian/sec.}$$

N is given. So, and also $s_c =$ ✓
calculate poles,

$$p_i = \pm s_c e^{j(N+2i+1)\pi/2N} \\ i = 0, 1, 2, \dots, N-1$$

$$p_0 = \pm s_c e^{j(2+1)\pi/2 \times 2} \\ = \pm s_c e^{j3\pi/4}$$

$$= \pm 6498.39 \left[\cos\left(\frac{3\pi}{4}\right) + j \sin\left(\frac{3\pi}{4}\right) \right]$$

$$= \pm 6498.39 \left[-0.707 + j 707 \right]$$

$$= \pm \left[-4594.36 + 4594.36j \right]$$

So,

$$p_0 = -4594.36 + 4594.36j \text{ and } 4594.36 - 4594.36j$$

Again,

$$P_2 = + s_2 e^{j(2+2+1)\pi/2 \times 2}$$

$$= + s_2 e^{j5\pi/4}$$

$$= + 6498.39 \left[\cos\left(\frac{5\pi}{4}\right) + j \sin\left(\frac{5\pi}{4}\right) \right]$$

$$P_1 = \frac{-4594.36 - j 4594.36}{4594.36 + j 4594.36} \text{ and}$$

So, take negative only.

$$P_0 = -4594.36 + j 4594.36$$

$$P_1 = -4594.36 - j 4594.36$$

Step-5] transfer of analog filter is.

$$H(s) = \frac{s_2^N}{(s - P_0)(s - P_1)}$$

$$= \frac{(6498.39)^2}{(s - P_0)(s - P_1)}$$

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$$[s - (-4594.36 + j 4594.36)] [s - (-4594.36 - j 4594.36)]$$

$$H(s) = \frac{(6498.39)^2 \cdot 42.22 \times 10^6}{(s + 4594.36 - j 4594.36)(s + 4594.36 + j 4594.36)}$$

$$= \frac{(6498.39)^2}{s^2 + 9188.72s + 1}$$

$$= \frac{1}{T} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right)$$

Unit - 4

FIR filter structure :-

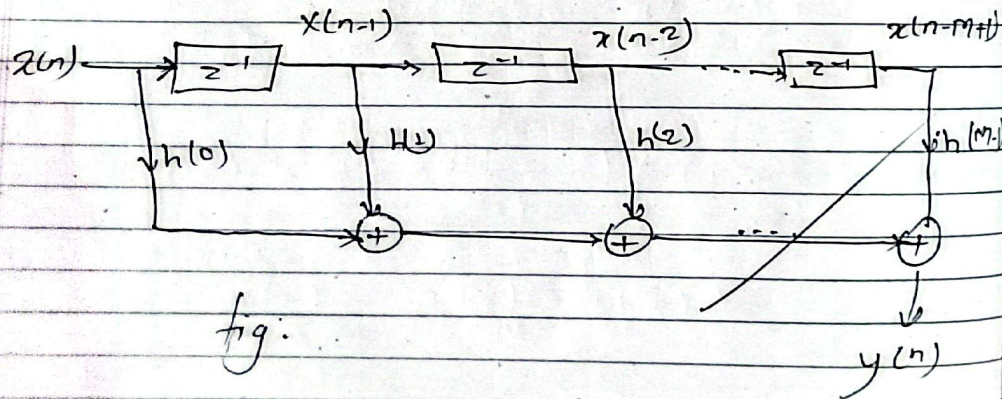
(1) Direct form structure:

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k)$$

Expanding,

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \dots + h(M-1)x(n-M+1)$$

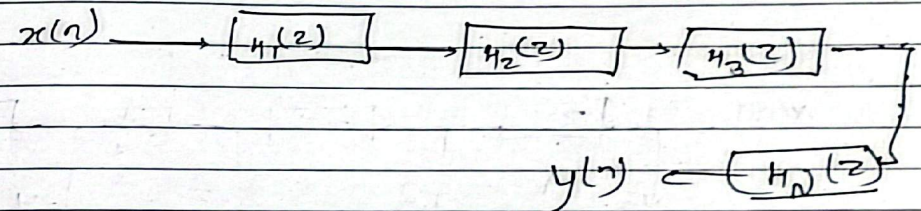


(2) Cascade form structure for FIR filter:

General eqⁿ:-

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \dots H_K(z)$$



$$y_k(n) = b_{k0} x_k(n) + b_{k1} x_k(n-1) + \dots$$

we know $h(z) = \frac{y(z)}{x(z)}$

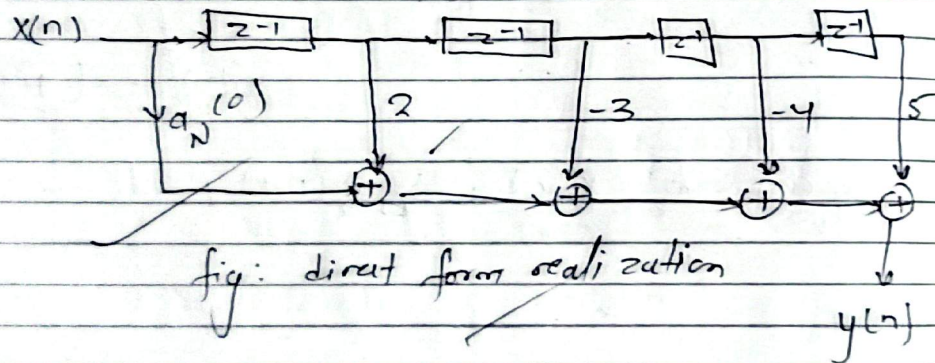
Q.7.

$$h(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

$$\frac{y(z)}{x(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

$$y(z) = x(z) + 2z^{-1}x(z) - 3z^{-2}x(z) - 4z^{-3}x(z) + 5z^{-4}x(z)$$

Now,



Q.8

$$h(z) = (1 + 2z^{-1} - z^{-2})(1 + z^{-1} - z^{-2})$$

here,

$$h(z) = h_1(z) \cdot h_2(z)$$

where, $h_1(z) = 1 + 2z^{-1} - z^{-2}$

$$h_2(z) = 1 + z^{-1} - z^{-2}$$

$$h_1(z) = \frac{y_1(z)}{x_1(z)}$$

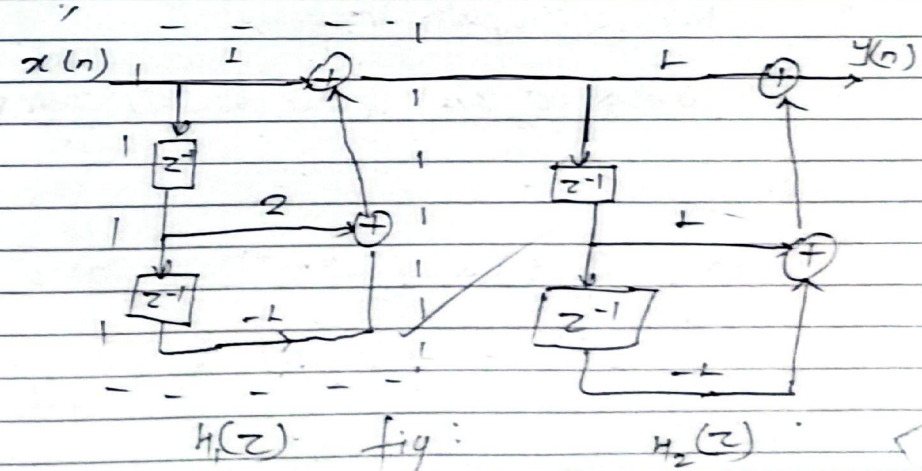
$$y_1(z) = x_1(z) + 2z^{-1}x_1(z) - z^{-2}x_1(z)$$

Again,

$$h_2(z) = \frac{y_2(z)}{x_2(z)}$$

$$y_2(z) = x_2(z) + z^{-1}x_2(z) - z^{-2}x_2(z)$$

Now,



Q.7 $H(z) = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$

$$= (1 + 2z^{-1}) \left(1 + \frac{1}{2}z^{-1} + 2z^{-2} \right)$$

✓ Lattice Structure:-

$$H(z) = \sum_{n=0}^{M=N} h(n) \cdot z^{-n} = \sum_{n=0}^N b_n z^{-n}$$

$$= \sum_{n=0}^N a_n(n) \cdot z^{-n}$$

$h(n) \rightarrow$ impulse response.

$b_n \rightarrow a_n(n) \rightarrow$ location of zeros.

or,

$$H(z) = a_N(0) + a_N(1)z^{-1} + a_N(2)z^{-2} + \dots + a_N(N) \cdot z^{-N}$$

$$a_N(0) = 1 \text{ (always)}$$

$a_N(n) = k_n$ where k_n is called lattice coefficient.

$a_N(0), a_N(1), a_N(2), \dots, a_N(N)$ is called direct form coefficient of FIR filter.

for lattice structure

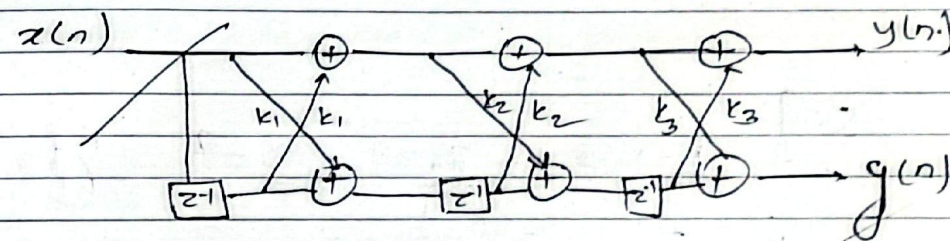


fig: lattice structure.

k_1, k_2, k_3 are called lattice coefficients.

$$k_1 = a_N(1)$$

$$k_2 = a_N(2)$$

$$k_3 = a_N(3)$$

Note: $|k_n| < 1$ for stability

Direct to lattice.

$$(i) \quad a_{m-1}(k) = \frac{a_m(k) - a_m(m) \cdot a_m(m-k)}{1 - a_m^2(m)}$$

where,

$$m = N, N-1, N-2, \dots$$

$$k = m-1, m-2, \dots$$

(ii) lattice to direct,

$$a_m(k) = a_{m-1}(k) + a_m(m) \cdot a_{m-1}(m-k)$$

Q.7. $H(z) = 1 + \frac{7}{9}z^{-1} + \frac{3}{5}z^{-2}$ (i)

Here,

$$N = 2$$

for FIR filter,

$$h(z) = \sum_{n=0}^N a(n) \cdot z^{-n}$$

$$= \sum_{n=0}^2 a(n) \cdot z^{-n}$$

$$= \frac{a(0)}{2} + \frac{a(1)}{2}z^{-1} + \frac{a(2)}{2}z^{-2} \quad (ii)$$

from eqn (i) and (ii),

$$\frac{a(0)}{2} = 1$$

$$\frac{a(1)}{2} = 7/9$$

$$\frac{a(2)}{2} = 3/5 = k_2$$

Direct to lattice coefficient conversion.

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m) \cdot a_m(m-k)}{1 - a_m^2(m)} \quad (iii)$$

$$N = 2, \quad m = 2, 1$$

for $m = 2,$
 $k = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(2-1)}{1 - a_2^2(2)}$$

$$a_1 = \frac{7/9 - \cancel{3/5} \times 7/9}{1 - (3/5)^2}$$

$$= 63/80$$

The lattice coefficients are,

$$k_1 = 1$$

$$k_2 = 3/5$$

Now,

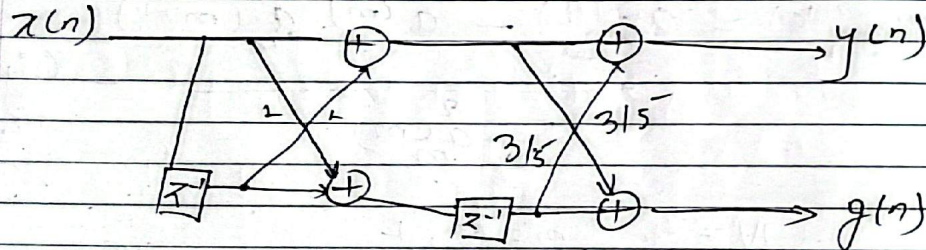


fig. two stage.

IIR structure:

Direct form - $\begin{bmatrix} I \\ II \end{bmatrix}$

General expression for IIR filter.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$$

order of IIR filter
 $(N \geq M)$

a_k = Poles, b_k = zeros.

where,

$$H(z) = H_1(z) \cdot H_2(z)$$

$$\text{where, } H_1(z) = \sum_{k=0}^M b_k z^{-k}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}}$$

Direct - II realization:

$$H(z) = \frac{\sum_{k=0}^M b_k \cdot z^{-k}}{1 + \sum_{k=1}^N a_k \cdot z^{-k}} \quad \text{--- (i)}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

where

$$H_1(z) = \frac{1}{1 + \sum_{k=1}^N a_k \cdot z^{-k}}$$

$$H_2(z) = \sum_{k=0}^M b_k \cdot z^{-k}$$

$$\begin{aligned} \text{8.7] } H(z) &= \frac{(1 + z^{-1})^2}{(1 - 0.5z^{-1})(1 + 0.75z^{-1})} \\ &= \frac{1 + 2z^{-1} + z^{-2}}{1 + 0.75z^{-1} - 0.5z^{-1} - 0.375z^{-2}} \\ &= \frac{1 + 2z^{-1} + z^{-2}}{1 + 0.25z^{-1} - 0.375z^{-2}} \quad \text{--- (ii)} \end{aligned}$$

General expression for IIR filter,

$$H(z) = \frac{\sum_{k=0}^M b_k \cdot z^{-k}}{1 + \sum_{k=1}^N a_k \cdot z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad \text{--- (ii)}$$

Comparing eqⁿ (i) and (ii) we get,

$$b_0 = 1, \quad b_1 = 2, \quad b_2 = 1$$

$$\text{and } a_1 = 0.25, \quad a_2 = -0.375$$

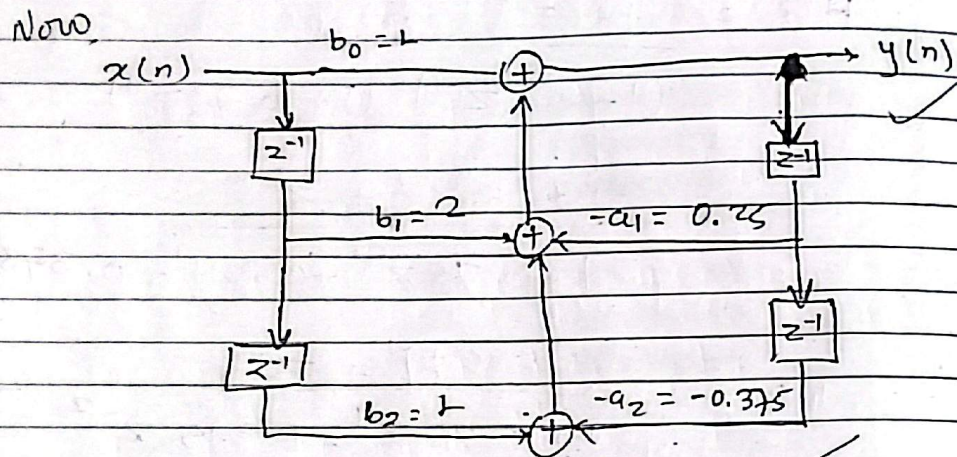


fig: Direct form-II realization

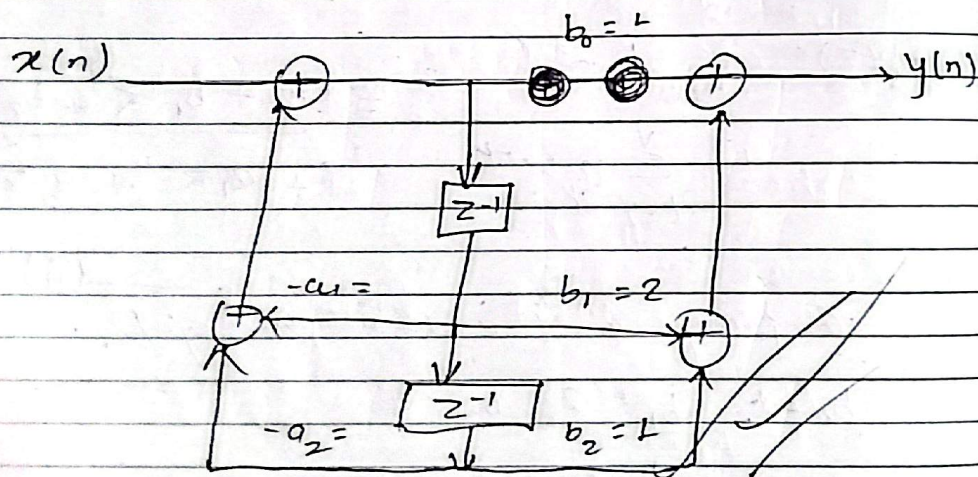


fig: Direct form-II realization.

Q.7. $H(z) = \frac{z(z+1)}{(z-1)(z+\frac{1}{4})}$

$$= \frac{z^2 + z}{(z^2 + \frac{1}{4}z - z - \frac{1}{4})}$$

$$= \frac{z^2 + z}{z^2 - \frac{3}{4}z - \frac{1}{4}}$$

$$= \frac{z^2}{z^2 - \frac{3}{4}z - \frac{1}{4}} \left(1 + \frac{1}{z}\right)$$

$$= \frac{z^2 \left(1 - \frac{3}{4z} - \frac{1}{4z^2}\right)}{1 - \frac{3z^{-1}}{4} - \frac{1}{4}z^{-2}}$$

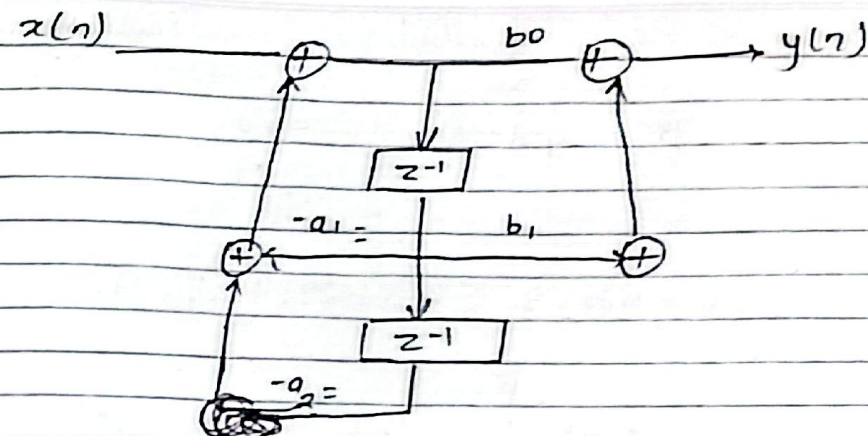
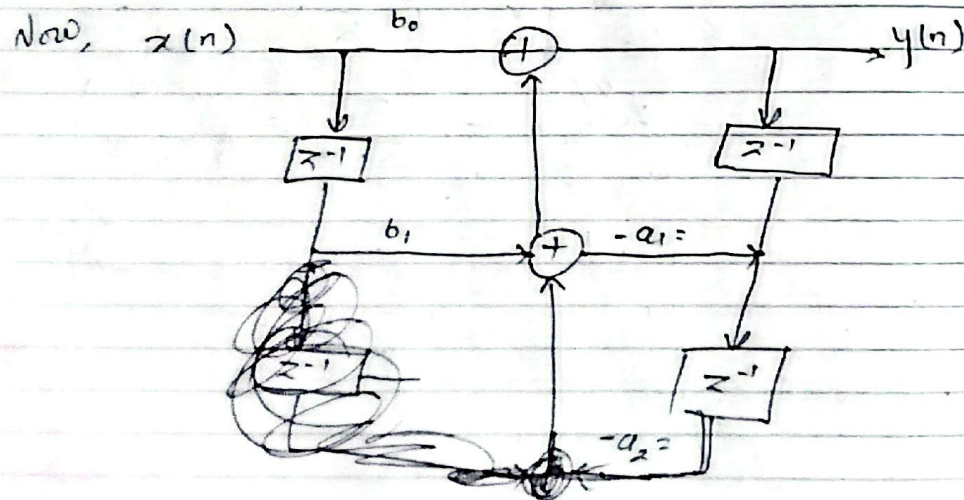
General expression,

$$H(z) = \frac{\sum_{k=0}^M b_k \cdot z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad \text{--- (ii)}$$

Comparing eqⁿ (i) and (ii) we get,

$$b_0 = 1, \quad b_1 = 1$$

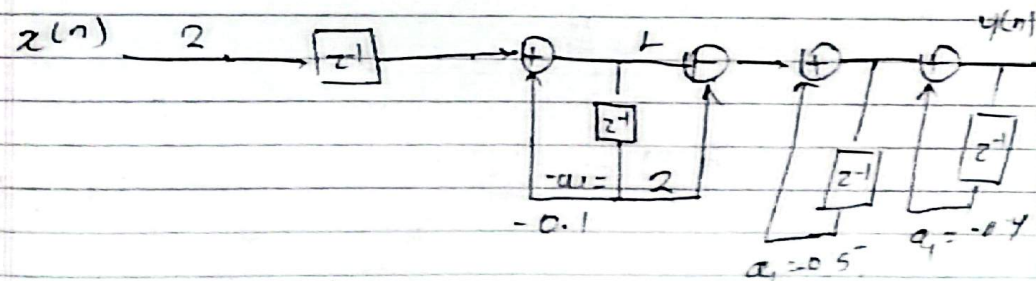
$$a_1 = -3/4, \quad a_2 = -1/4$$



Cascade realization:

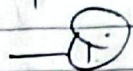
$$H(z) = H_1(z) \cdot H_2(z) \cdot \dots \cdot H_K(z)$$

using general expression.

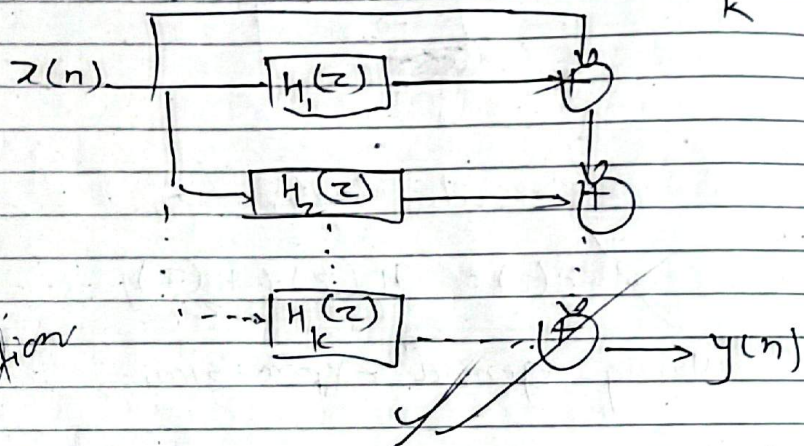


parallel form realization:

Use. general expression.



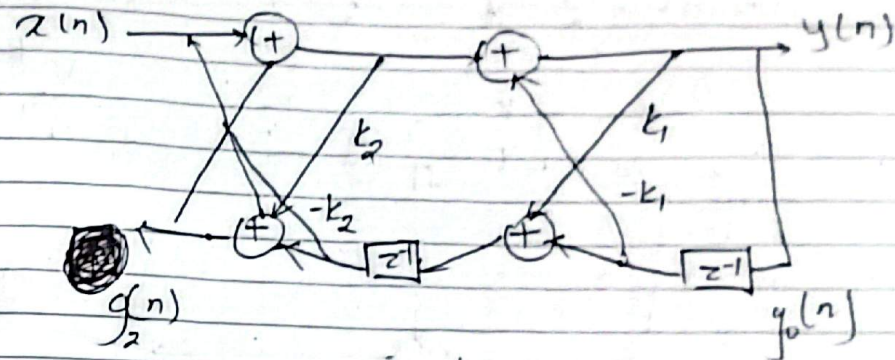
$$H(z) = C + \frac{H_1(z)}{C} + \frac{H_2(z)}{C} + \dots + \frac{H_K(z)}{C}$$



use. Partial fraction

lattice:

$$H(z) = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}}$$



lattice ladder structure.

$$H(z) = \frac{B_M(z)}{A_N(z)} = \frac{\sum_{k=0}^N b_m(k) \cdot z^{-k}}{1 + \sum_{k=1}^N a(k) \cdot z^{-k}}$$

$$M=N$$

$$c_m = b_m - \sum_{i=m+1}^M c_i a_i (i-m)$$

$$m = M, M-1, \dots, 0$$

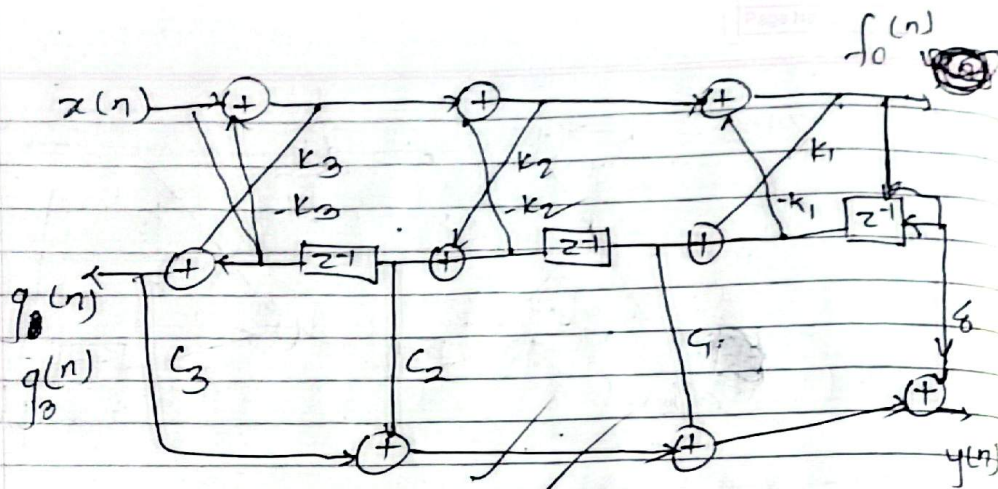


fig: Lattice ladder structure.

$$N=M=3 \quad m=3, 2, 1, 0$$

$$c_m = b_m - \sum_{i=m+1}^M c_i a_i^{i-m}$$

$$c_3 = b_3 = 1.$$

$$\begin{aligned} c_2 &= b_2 - \sum_{i=3}^3 c_i a_i^{i-2} \\ &= b_2 - c_3 a_3^{1} \\ &= 2 - 1 \cdot \left(\frac{13}{4}\right) = \frac{5}{4} \end{aligned}$$

$$= 2 - 1 \cdot \left(\frac{13}{4}\right) = \frac{5}{4}$$

$$\begin{aligned} b_0 &= k_1 \\ b_1 &= 2 \\ b_2 &= 2 \\ b_3 &= 1 = c_3 \end{aligned}$$

Q. 4.(b)

Here,

$$y(n) = -\frac{3}{4} y(n-1) + \frac{1}{4} y(n-2) + x(n) + \frac{1}{2} x(n-1)$$

Taking z-transform on both side,

$$Y(z) = -\frac{3}{4} z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) + X(z) + \frac{1}{2} z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-1}}{1 + \frac{3}{4} z^{-1} - \frac{1}{4} z^{-2}} \quad \text{--- (i)}$$

The general eqⁿ is.

$$N=2$$

$$H(z) = \frac{\sum_{k=0}^N b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} \quad \text{--- (ii)}$$

Comparing eqⁿ (i) and (ii). we get,

$$b_0 = 1, \quad b_1 = 1/2$$

$$a_2(1) = 3/4, \quad a_2(2) = -1/4 = k_2$$

convert the direct form to lattice coefficient.

$$a_{m-1}(k) = \frac{a_2(k) - a_2(m) \cdot a_2(m-k)}{1 - a_2(m)^2} \quad \text{--- (iii)}$$

Where,

$$m = N, N-1, N-2, \dots$$

$$N = 2,$$

$$m = 2, 1$$

$$k = m-1, m-2, \dots$$

$$m = 2, \quad k = 1.$$

eqn (iii) becomes,

$$\begin{aligned} a_1(1) &= \frac{a_2(1) - a_2(2) \cdot a_2(1)}{1 - a_2(2)^2} \\ &= \frac{3/4 - (-1/4) \times (3/4)}{1 - (-1/4)^2} \end{aligned}$$

$$a_1(1) = 1.$$

$$\text{So, } k_1 = 1, \quad k_2 = -1/4.$$

Again, for ladder structure.

$$c_m = b_m - \sum_{l=m+1}^M c_l a_l(1-m) \quad \text{--- (iv)}$$

where,

$$M = M, M-1, M-2, \dots, 0$$

$$m = 2, 1, 0$$

$$c_2 = b_2 = 0$$

$$\therefore c_1 = b_1 - \sum_{l=2}^2 c_l a_l(2-1)$$

$$= b_1 - 0$$

$$\therefore c_1 = 1/2$$

$$\therefore c_0 = b_0 - \sum_{l=1}^2 c_l a_l(1-0)$$

$$= 1 - c_1 a_1(1) = 1 - \frac{1}{2} \times 1 = \frac{1}{2}$$

Now,

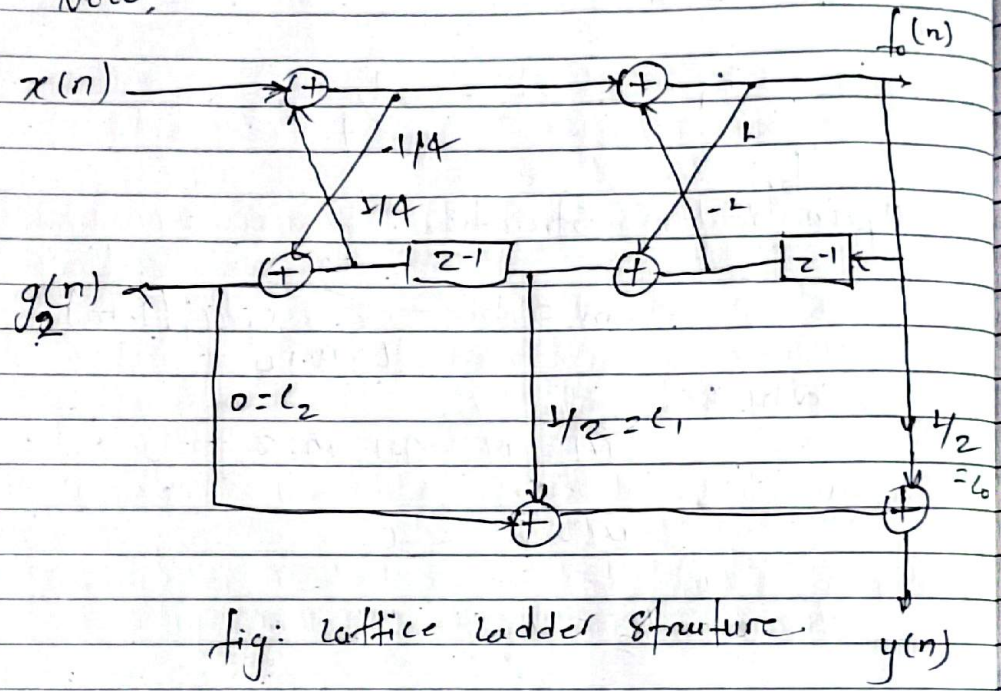


fig: lattice ladder structure

Short Notes :-

a) Recursive system	Non-recursive system
→ It depends on past inputs as well as past outputs.	→ It depends only on inputs.
→ In recursive system, feedback is present.	→ In non-recursive system, no feedback present.
→ It may be unstable	→ It is always stable.
→ It equation: includes $y(n-k)$ terms	→ It equation: includes only $x(n-k)$ terms.
→ Example: IIR filter	→ Example: FIR filter.

aliasing.

b) Circular Convolution	Linear Convolution.
→ It's output length is N	→ It's output length is $N + M - 1$.
→ It is types of periodic signal	→ It is types of non-periodic signal
→ It's aliasing is possible.	→ The aliasing is not possible.
→ It uses is time in DFT/FFT	→ It uses in time domain system.
→ padding padding required to match linear result	→ padding padding is not required.

specs

* Chebyshev filter

Elliptic filters

→ It's ripple is passband (Type-I) or stopband (Type-II)

→ It's ripple is both. Passband and stopband.

→ It's transition is sharp

→ It's transition is very sharp.

→ It's complexity is moderate

→ It's complexity is high

→ filter order is higher than elliptic

→ It's filter order is lowest for same specs.

→ It's design is simpler

→ It's design is more complex

→ It's phase response is better than elliptic

→ It's phase response is poor

* DTFS

DTFT

→ It is type of periodic signal

→ It is type of non-periodic / aperiodic signal

→ It's frequency spectrum is discrete

→ Continuous

→ Its representation is finite sum

→ Its representation is infinite integral

Advantage → Simple and systematic.
→ Easy to implement using FFT algorithm

→ It's stands for Discrete time Fourier series.

→ It's stands for Discrete time Fourier transform

→ It is used in analysis of periodic sequences.

→ It is used in general signal analysis.

* Frequency Sampling

→ The frequency sampling method is used in DSP to design FIR filters by sampling the desired frequency response.

→ The ideal response $h(e^{j\omega})$ is sampled at equally spaced frequencies.

→ formula:
$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j \frac{2\pi kn}{N}}$$

→ Application:

→ Digital filter design
→ Spectrum analysis

Disadvantage: → Ripple effects may occur.
→ Not ideal for sharp transitions

*
a) DTFT and properties:-

→ If the $x[n]$ is a non-periodic signal (sequence) then its corresponding Fourier transform is given by $X(e^{j\omega})$

In mathematically,

$$x[n] \xrightarrow{\text{DTFT}} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Properties:-

1) Linearity: It states that if $x_1[n] \xrightarrow{\text{DTFT}} X_1(e^{j\omega})$ and $x_2[n] \xrightarrow{\text{DTFT}} X_2(e^{j\omega})$ then

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] \xrightarrow{\text{DTFT}} \alpha_1 X_1(e^{j\omega}) + \alpha_2 X_2(e^{j\omega})$$

2) Time shifting property: It states that if $x[n] \xrightarrow{\text{DTFT}} X(e^{j\omega})$ then

$$x[n - n_0] \xrightarrow{\text{DTFT}} e^{-j\omega n_0} X(e^{j\omega})$$

3) Differentiation property: It states that if $x[n] \xrightarrow{\text{DTFT}} X(e^{j\omega})$ then

$$\frac{d}{dn} x[n] \xrightarrow{\text{DTFT}} j\omega X(e^{j\omega})$$

4) Convolution property: It states that if $x_1[n] \xrightarrow{\text{DTFT}} X_1(e^{j\omega})$ and $x_2[n] \xrightarrow{\text{DTFT}} X_2(e^{j\omega})$ then

$$x_1[n] \otimes x_2[n] \xrightarrow{\text{DTFT}} X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$$

* Remez Exchange Algorithm:

→ The Remez Exchange Algorithm is also called Parks - McClellan algorithm is an efficient method for designing optimal FIR filters with minimum maximum error.

→ It uses iterative process to adjust frequency points.

→ It is based on the alternation theorem.

Advantages:

→ It is best approximation.

→ Efficient for sharp transition filters.

Disadvantages:

→ It is more complex than window method.

→ It requires iterative computation.

* Kaiser window:

→ The Kaiser window is a flexible window function used in FIR filter design to control main lobe width and side lobe attenuation.

→ It uses a parameter β to adjust the shape of the window.

→ Window function,

$$w(n) = \frac{I_0(\beta)}{I_0(\beta)} \left[1 - \left(\frac{n - \alpha}{\alpha} \right)^2 \right]^{\beta/2} \quad \text{for } 0 \leq n \leq M$$

where,

$$\alpha = \frac{M}{2}$$

$$\beta = \begin{cases} 0.1102 (A - 8.7) & \text{for } A > 50 \\ 0.5842 (A - 21)^{0.4} + 0.07886 (A - 21) & \text{for } 21 \leq A \leq 50 \\ 0 & \text{for } A < 21 \end{cases}$$

$$M = \frac{A - 8}{2.285 \Delta\omega}$$

where $\Delta\omega = \omega_s - \omega_p$.

and

$$A = -20 \log_{10}(S)$$

→ It is flexible and adjustable.

→ It is widely used in practical IIR design.

→ It is slightly more complex.

2024- Spring

Q.N. 2.b] Definition →

In note:

Here, $X(z) = \log(1 + az^{-1})$: ROC: $|z| > |a|$

$$\frac{d}{dz} X(z) = \frac{1}{1 + az^{-1}} (-az^{-2})$$

$$\frac{d}{dz} X(z) = \frac{-az^{-2}}{1 + az^{-1}}$$

$$-z \frac{d}{dz} X(z) = \frac{az^{-1}}{(1 + az^{-1})} \quad \text{ROC: } |z| > |a|$$

Taking inverse z-transform both side.

$$nx[n] = a(-a)^{n-1}u(n-1) \quad \left(\begin{array}{l} \because a^n u(n) \\ \longleftrightarrow \frac{1}{(1 - az^{-1})} \end{array} \right)$$

$$x[n] = \frac{1}{n} a(-a)^n \frac{1}{-a} \cdot u(n-1) \quad \left(\begin{array}{l} \longleftrightarrow \frac{1}{(1 - az^{-1})} \end{array} \right)$$

$$x[n] = -a \frac{(-a)^n u(n-1)}{n} \quad \# \quad \left(\begin{array}{l} (-a)^n u(n) \\ \longleftrightarrow \frac{1}{(1 + az^{-1})} \end{array} \right)$$

e

$$\left[\begin{array}{l} a(-a)^{n-1}u(n-1) \longleftrightarrow \frac{a}{(1+az^{-1})} \\ a(-a)^{n-1}u(n-1) \longleftrightarrow \frac{az^{-1}}{(1+az^{-1})} \end{array} \right]$$

$|z| > |a|$

4.b] IIR filter Design approaches:

→ window Design method

An ideal impulse response $h_d(n)$ is multiply by a finite window function $w(n)$.

→ frequency sampling method:

The desired frequency response is sampled and an inverse Discrete fourier transform (IDFT) is used to find the filter coefficients.

→ optimal / Equiripple (Parks-McClellan):
uses the Remez exchange algorithm
to minimize the maximum error
between desired and actual frequency
response.

Designing an FIR filter using a rectangular
window.

The rectangular window is simplest windowing
technique. defines a rectangular pulse
of length M .

Steps:

1. Define the ideal filter.

i.e. $H_d(e^{j\omega})$, such as lowpass filter
with cutoff ω_c .

2. → Compute ideal impulse response.

$$\text{i.e. } h_d(n) = \frac{\sin(\omega_c n)}{\pi n}$$

3. Define the window function.

$$\text{i.e. } w(n) = 1 \text{ for rectangular window}$$
$$0 \leq n \leq M-1$$

4. multiply to find the filter coefficient.

$$\text{i.e. } h(n) = h_d(n) \cdot w(n)$$

5. shift for causality:

$$\text{i.e. } h(n) = h_d\left(n - \frac{M-1}{2}\right) \cdot w(n)$$

2025-SP.
(New).

4-b]

The Impulse Invariant Method (IIM) is a technique used to design infinite impulse response (IIR) digital filters.

Derivation of IIM:

The goal is to find a digital transfer function $H(z)$.

Step-1] Define the analog Impulse Response

i.e. $h_a(s)$

Use Partial Fraction Expansion

i.e. $h_a(s) = \sum_{i=1}^N \frac{A_i}{(s-p_i)}$

Where.

$p_i \rightarrow$ poles of analog filter.

$A_i \rightarrow$ residues.

Step-2] Inverse Laplace transform:

i.e. $h_a(t) = \mathcal{L}^{-1}\{H_a(s)\}$
 $= \sum_{i=1}^N A_i e^{p_i t}, t \geq 0$

Step-3] Sampling:

i.e. $h(n) = h_a(t) \big|_{t=nT}$
 $= \sum_{i=1}^N A_i e^{p_i nT}$

Step-4] Take z-transform.

i.e. $H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$
 $= \sum_{i=1}^N \frac{A_i}{1 - e^{p_i T} z^{-1}}$

Mapping from s-plane to z-plane.

→ The mapping is defined by the relationship

$$z = e^{sT}$$

so,

→ Left Half of s-plane ($\text{Re}(s) < 0$):

maps inside the unit circle ($|z| < 1$).

→ Imaginary Axis ($\text{Re}(s) = 0$): maps onto the unit circle ($|z| = 1$).

→ Right Half of s-plane ($\text{Re}(s) > 0$): maps outside the unit circle ($|z| > 1$).

Q.N. 5(b)

FIR filter design using the Kaiser window involves determining specifications.

ω_s → Stopband edge frequency

ω_p → Passband edge frequency.

β → Kaiser parameter

$\Delta\omega$ → transition bandwidth = $\omega_s - \omega_p$

α → Kaiser parameter

A → ripple

ω_c / f_c → cutoff frequency

Step-1.] Calculate Kaiser parameter

$$\gamma = \frac{M-1}{2}, \quad M \rightarrow \text{length of filter.}$$

$$\beta = \begin{cases} 0.1102(A-8.7) & \text{for } A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & \text{for } 21 \leq A \leq 50 \\ 0 & \text{for } A < 21 \end{cases}$$

Where.

$$A = -20 \log_{10}(\delta)$$

Where $\delta \rightarrow$ minimum of δ_1 and δ_2

Step-2] determine the order (M).

$$M = \frac{A-8}{2.285 \Delta\omega}$$

$$\text{where } \Delta\omega = \omega_s - \omega_p$$

Step-3] Compute the ideal impulse response.

$$\text{i.e. } h_d(n) = \frac{1}{2\pi} \int_{-\omega_s}^{\omega_s} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

Step-4] Compute the Kaiser window function.

$$w(n) = \frac{I_0 \left\{ \beta \left[1 - \left(\frac{n-\alpha}{\alpha} \right)^2 \right]^{1/2} \right\}}{I_0(\beta)} \quad \text{for } 0 \leq n \leq M$$

Step-5] finally calculate the actual FIR filter coefficient

$$\text{i.e. } h(n) = h_d(n) \cdot w(n) \quad \text{for } 0 \leq n \leq M$$

Z-transform:

→

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \quad \text{--- (i)}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} + \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

non-causal causal

→

Roc: Region of Convergence:

Q.7. $u(n)$.

We know.

$$u(n) = 1 \quad ; \quad \text{for } n \geq 0$$

0 ; otherwise.

$$Z\{u(n)\} = \sum_{n=-\infty}^{\infty} u(n) \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} 1 \cdot z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \dots$$

$$= 1 + (z^{-1})^1 + (z^{-1})^2 + (z^{-1})^3 + \dots$$

$$= \frac{1}{1 - z^{-1}} \quad \# \quad \left(\frac{a}{1 - ar} \right)$$

for Roc, $|z^{-1}| < 1$.

$$\therefore \left| \frac{1}{z} \right| < 1$$

$\therefore |z| > 1$. required Roc.

i) $u[n] = \frac{1}{1-z^{-1}}$

ii) $u[-n-1] = \frac{-1}{1-z^{-1}}$

Properties:

i). Linearity:

It states that, if $x_1[n] \xrightarrow{Z} X_1(z)$ and $x_2[n] \xrightarrow{Z} X_2(z)$ then

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] \xrightarrow{Z} \alpha_1 X_1(z) + \alpha_2 X_2(z)$$

where α_1 and α_2 are constant.
arbitrary.

2) Time shifting property:

It states that, if $x[n] \xrightarrow{Z} X(z)$ then

$$x[n-n_0] \xrightarrow{Z} z^{-n_0} X(z).$$

where n_0 is constant time domain.

iii) Scaling property:

It states that, if $x[n] \xrightarrow{Z} X(z)$ then
an $x[n] \xrightarrow{Z} X(a^{-1}z)$

where a is constant.

iv) Differentiation property:-

It states that, if $x[n] \xrightarrow{Z} X(z)$
then $n \cdot x[n] \xrightarrow{Z} -z \frac{d}{dz} X(z)$

v) Convolution property:

It states that, if $x_1[n] \xrightarrow{Z} X_1(z)$ and $x_2[n] \xrightarrow{Z} X_2(z)$ then $x_1[n] \otimes x_2[n] \xrightarrow{Z} X_1(z) \cdot X_2(z)$

$a^n u[n] \rightarrow \frac{1}{1 - az^{-1}}$
 $u[n] \rightarrow \frac{1}{1 - z^{-1}}$ / $u[-n] \rightarrow \frac{1}{1 - z}$
~~Numerical~~ for partial fraction.

(I) $\frac{1}{(s+a)(s+b)} \rightarrow \frac{A}{(s+a)} + \frac{B}{(s+b)}$

(II) $\frac{1}{(s+a)(s^2+as+b)} \rightarrow \frac{A}{(s+a)} + \frac{Bs+C}{(s^2+as+b)}$

for Numerical:

signal	z-transform	ROC
1. $\delta[n] =$	1	
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
3. $a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
4. $na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
5. $-a^n u[-n-1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
6. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $

7. $(\cos \omega_0 n) u[n] \rightarrow \frac{1 - z^{-1} \cos \omega_0}{1 - 2z^{-1} \cos \omega_0 + z^{-2}} \quad |z| > 1$

8. $\sin \omega_0 n u[n] \rightarrow \frac{z^{-1} \sin \omega_0}{1 - 2az^{-1} \cos \omega_0 + a^2 z^{-2}} \quad |z| > |a|$

9. $(a^n \cos \omega_0 n) u[n] \rightarrow \frac{1 - az^{-1} \cos \omega_0}{1 - 2az^{-1} \cos \omega_0 + a^2 z^{-2}} \quad |z| > |a|$

10. $(a^n \sin \omega_0 n) u[n] \rightarrow \frac{az^{-1} \sin \omega_0}{1 - 2az^{-1} \cos \omega_0 + a^2 z^{-2}} \quad |z| > |a|$

2024 'F'
Numerical:

2.6) find z-transform and ROC of
 $x[n] = 2\left(\frac{5}{6}\right)^n u[n-1] + \left(\frac{1}{2}\right)^{2n} u[n]$.

Taking first term, $x_1[n] = 2\left(\frac{5}{6}\right)^n u[n-1]$

~~using~~ using time shifting property.

$$x_1[n] = 2\left(\frac{5}{6}\right) \left(\frac{5}{6}\right)^{n-1} u[n-1]$$

we know,
 $x[n-k] \xrightarrow{z} z^{-k} X(z)$

i.e. $a^n u[n] \xrightarrow{z} \frac{z}{z-a} = \frac{1}{1-az^{-1}}$

$$X_1(z) = \frac{5}{3} \cdot z^{-1} \left(\frac{z}{z - 5/6} \right) = \frac{5/3}{z - 5/6}$$

Again
 Taking, second term.

$$x_2[n] = \left(\frac{1}{2}\right)^{2n} u[n]$$

$$x_2[n] = \left(\frac{1}{4}\right)^n u[n]$$

• we know,

$$a^n u[n] \xrightarrow{z} \frac{z}{z-a}$$

So

$$X_2(z) = \frac{z}{z - \frac{1}{4}}$$

Now, combine both,

$$X(z) = \frac{5/3}{z - 5/6} + \frac{z}{z - 1/4}$$

$$= \frac{5/3 \cdot z^{-1}}{1 - \frac{5}{6}z^{-1}} + \frac{1}{1 - \frac{1}{4}z^{-1}}$$

Again, ROC:

For first term, ROC: $|z| > 5/6$

for second term, ROC: $|z| > 1/4$

$$u(n-1) = \frac{1}{1-z^{-1}}$$

Pole: $|z| < 1$



2.b]

Here Given $x(n) = a^n u(n)$

By scaling property of z-transform.

We know,

$$a^n u(n) \xrightarrow{z} \frac{1}{1-az^{-1}}$$

So,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} u(n) \cdot (a^{-1}z)^{-n}$$

$$= \sum_{n=0}^{\infty} 1 \cdot (a^{-1}z)^{-n}$$

$$= 1 + (a^{-1}z)^{-1} + (a^{-1}z)^{-2} + (a^{-1}z)^{-3} + \dots$$

$$= \frac{1}{1 - (a^{-1}z)^{-1}} = \frac{1}{1 - az^{-1}}$$