

Assignment -

Sub: Simulation and Modeling :

- 1.a Describe in brief the different types of simulation models. Taking an example of any realistic simulation problem, illustrate the concepts of system boundary, environment, entities, endogenous and exogenous.

⇒ Types of Simulation Models.

↳ Continuous and Discrete System

→ A system in which the state variable changes continuously over time is called Continuous System.

→ Example : Aircraft system.

→ A system in which state variable changes only at discrete set of points in time is called discrete system.

→ Example: A banking system.

2. Static and Dynamic system

↳ Static models represent a system at one fixed point in time.

↳ Example: Monte Carlo Method.

↳ Dynamic models represents how a system evolves over time

↳ Example. Queuing system.

3. Deterministic and Stochastic system.

↳ The system in which the outcomes of an activity can be completely described in terms of it's input are called deterministic system.

↳ The system in which the outcomes of an activity can not be completely described in terms of it's input or vary over various possible outcomes is called stochastic system.

4. Open and Closed System.

↳ A system for which there is no exogenous activity is said to be closed system otherwise it is called open system.

Illustrative Example: Bank Queuing System.

- System boundary: The physical/logical limit of what is modeled. Here, the bank hall including the entrance, teller counters and waiting area. Anything outside this is excluded from detailed modeling.
- Environment: Factors outside the system boundary that affect it but aren't modeled in detail. Example weather, public holidays, salary day, which influence how many customers arrive.
- Entities: The objects that flow through or act within the system. Eg. Customers, tellers, the queue itself.
- Endogenous Activities: Activities that occur within the system boundary and

are directly modeled. Ex. a teller serving a customer, a customer waiting in line.

→ Exogenous activities: Activities originating outside the system that affect it. eg. the arrival process of customers etc.

1.6] Solve the following Monte Carlo simulation. Use 15 iteration to generate the number. Also estimate the error percentage.

$$I = \int_0^{\pi/2} \sin x \cos x \, dx. \quad 0 \leq x \leq \pi/2$$

⇒ Here,

$$y = f(x) = \sin x \cos x \quad 0 \leq y \leq 1/2$$

$$a = 0$$

$$b = \pi/2$$

$$\begin{aligned} \text{Area of Curve (A)} &= (b-a) \times c \quad [\because c = 1/2] \\ &= \frac{\pi}{2} \times \frac{1}{2} \\ &= \frac{\pi}{4} \end{aligned}$$

Now, 15 iteration :-

Iteration	x	y	$f(x) = \sin x \cos x$	$y \leq \sin x \cos x$
1	0.15	0.08	0.148	Accept
2	0.32	0.25	0.297	Accept
3	0.45	0.35	0.392	Reject
4	0.6	0.48	0.466	Accept
5	0.72	0.30	0.496	Accept
6	0.85	0.42	0.46	Accept
7	0.95	0.18	0.473	Reject
8	1.05	0.46	0.433	Accept
9	1.15	0.22	0.372	Reject
10	1.25	0.41	0.299	Accept
11	1.35	0.15	0.214	Accept
12	1.42	0.12	0.148	Accept
13	1.48	0.09	0.090	Reject
14	1.52	0.07	0.051	Accept
15	1.56	0.01	0.011	Accept

Here, Accepted (M) = 11

Rejected ~~4~~ = 4

Total (N) = 15

Now, By Monte Carlo Method,

$$\hat{I} = A \times \frac{M}{N}$$

$$= \frac{11}{4} \times \frac{11}{15}$$

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$$\therefore I = 0.576$$

again,

Exact solution is,

$$I = \int_0^{\pi/2} \sin x \cos x dx = 0.5$$

$$\% \text{ Error} = \left| \frac{\text{Exact} - \text{approximate}}{\text{Exact}} \right| \times 100\%$$

$$= \left| \frac{0.5 - 0.576}{0.5} \right| \times 100\%$$

$$= 15.2\%$$

2.07. Compare simulation and Analytical method with example.

⇒ Simulation method Analytical method.

- | | |
|--|---|
| → It uses a computer model to imitate the behavior of a real system. | → It uses mathematical equations to obtain the exact solutions. |
| → Gives approximate results. | → It gives exact result (if solvable). |
| → It suitable for complex system with randomness | → It suitable for simple systems with known mathematical relationships. |
| → Can include uncertainty and random variables. | → Usually assumes deterministic conditions. |
| → It time consuming due to repeated simulation runs. | → faster once the mathematical model is formulated. |
| → It does not always requires simplifying assumptions. | → often requires simplifying assumptions. |
| → It useful when analytical | → limited to problem that can be solve |

Solution are difficult
or impossible.

mathematically.

→ Example: Traffic simulation,
queuing systems,
weather forecasting.

→ Example: Solving
 $x^2 + 5x + 6 = 0$, calculating
 $\int_0^1 x^2 dx$.

Example: Analytic method,

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1$$

$$= \frac{1}{3}$$

Simulation method,

random number $x \in [0, 1]$. then.

$$\bar{I} = \frac{1}{N} \sum_{i=1}^N x_i^2$$

$$= \frac{1}{3}$$

2.6] Define differential equation. Explain why it is required in continuous system simulation. Explain the types of differential equation with example.

⇒ A differential equation is an equation that relates an unknown function with one or more of its derivatives. It describes how a variable changes with respect to another variable.

General form, i.e. $\frac{dx}{dt} = f(x, t)$

$$f(x, y, y', y'', \dots, y^{(n)}) = 0$$

Where,

$x \rightarrow$ independent variable.

$y \rightarrow$ dependent variable.

$y', y'' \rightarrow$ first and second derivatives of y .

Why it is required in continuous system simulation :-

Continuous systems change continuously over time. Their behavior is described by differential equations.

- Models continuous changes in system variable
- predict system behaviour over time
- helps analyze dynamic system.
- Used in engineering, economics, biology and physics

Example:

- Population growth
- motion of vehicles
- predator - prey model
- water level in a tank etc.

Types of Differential Equations:-

a. Ordinary Differential Equation (ODE):
An equation involving derivatives with respect to one independent variable.

$$\text{i.e. } \frac{dy}{dt} = 3y.$$

b. Partial Differential Equation (PDE):-
An eqⁿ involving partial derivatives with respect to two or more independent variables.

$$\text{i.e. } \frac{\partial y}{\partial t} = k \frac{\partial^2 y}{\partial x^2}$$

c. Linear differential Equation:

The dependent variable and its derivatives appear only to the first power, no product between them.

$$\text{i.e. } \frac{dx}{dt} = 3x + 5$$

d. Non-linear differential Equation :-

It involves product, powers or non-linear function of the dependent variable.

$$\text{i.e. } \frac{dx}{dt} = ax - bxy$$

or,

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 0$$

3.a) Derive and write the GMP-III program for predator-prey model.

⇒ Let x = prey population, y = predator population.
prey grow naturally at rate ' a ', but are reduced by predation at proportional to encounters ' bxy '

$$\text{i.e. } \frac{dx}{dt} = ax - bxy$$

Again, predators die naturally at rate ' c ', but grow from consuming prey at rate proportional to encounters ' $d \cdot xy$ '

$$\text{i.e. } \frac{dy}{dt} = -cy + dxy$$

Where,

$a \rightarrow$ prey growth rate

$b \rightarrow$ predation rate coefficient

$c \rightarrow$ predator death rate

$d \rightarrow$ predator growth rate from predation.

CSMP-III Program :-

TITLE PREDATOR-PREY MODEL SIMULATION.

* State equations

$$X = \text{Intgrl}(X_0, DX)$$

$$Y = \text{Intgrl}(Y_0, DY)$$

* Derivative equation:-

$$DX = A * X - B * X * Y$$

$$DY = -C * Y + D * X * Y$$

* Parameters

$$\text{PARAM } A = 1.0, B = 0.5, C = 0.75, D = 0.25$$

* Initial Conditions

$$\text{CONST } X_0 = 10.0, Y_0 = 5.0$$

* Timing Control

$$\text{TIMER } \text{INTIM} = 50.0, \text{DELT} = 0.01, \\ \text{PRDEL} = 1.0$$

METHOD RKS

* Output

```
PRT PLOT X, Y  
LABEL PRE(X) VS PREDATOR(Y) POPULAT  
-ION OVER TIME.
```

```
END  
STOP  
END JOB.
```

3.6] What is the significance of incorporating delayed calls in a telephone system simulation & provide a detailed description of the simulation involving these delayed calls and how they impact the overall system performance.

⇒ A delayed call is a telephone call that cannot be connected immediately because all communication channels or operators are busy. Instead of being blocked, the call waits in a queue until a channel becomes available.

Significance :-

- Reduce call blocking
- Improve customer satisfaction.
- Models real telephone exchanges accurately
- helps determine the required number of channels.
- Measure waiting time and queue length
- Improve resource utilization.
- helps optimize network performance.

#. Simulation of Delayed Calls.

These simulation follows these steps :-

1. Call arrive randomly (usually following a Poisson process)
2. If a telephone line is free, the call is connected immediately.
3. If all lines are busy, the call joins a waiting queue.
4. When a line becomes free, the first waiting call is connected (FIFO).
5. After service, the line becomes available for next waiting call.

6. Statistics such as waiting time, Queue length, and utilization are recorded.

It impacts the overall system performance.

→ Determine the optimum number of telephone lines.

→ Reduces customer waiting time

→ Improve network efficiency.

→ Assists in planning and expansion of telephone systems.

→ Enables testing of different traffic conditions without affecting the real network.

4.a] Explain different gathering statistics the counter and summary statistics and the measuring utilization and occupancy.

⇒ Gathering Statistics :-

a) Counter Statistics:

Counter statistics count the number of times an event occurs during the simulation.

Example :-

- Number of arrived customers.
- Number of customers served.
- Number of customers waiting.
- Number of jobs completed.

Purpose :-

- To measure the event frequencies.
- To calculate throughput and system performance.

b) Summary Statistics:

Summary statistics provides overall information about the simulation results.

They include :-

- Average waiting time
- Average queue length
- maximum waiting time
- Average service time
- Standard deviation.

Purpose :-

- To Summarize the system performance.
- To Compare different simulation models.

#. Utilization :

utilization is the proportion of time a resource (server, machine, telephone line) is busy.

Formulae,
$$\text{Utilization} = \frac{\text{Busy Time}}{\text{Total Simulation Time}}$$

or,
$$\text{Utilization}(\%) = \frac{\text{Busy time}}{\text{Total Time}} \times 100$$

#. Occupancy:

Occupancy is the average number of entities occupying a resource or system during the simulation.

formula,

$$\text{Occupancy} = \frac{\text{Total occupied time}}{\text{Total simulation time}}$$

4.6] Use the Kolmogorov-Smirnov test to determine whether the given sequence is a sequence of uniform random number at 95% level of significance.

0.26, 0.66, 0.75, 0.58, 0.64, 0.98, 0.31, 0.90
0.88, 0.65.

⇒ Here,

Given random number

0.26, 0.66, 0.75, 0.58, 0.64, 0.98
0.31, 0.90, 0.88, 0.65

number of observation $n = 10$

Arrange in Ascending Order.

Step 1: R_i : 0.26, 0.31, 0.58, 0.61, 0.65, 0.66
 0.75, 0.88, 0.90, 0.98

$$D^+ = \frac{i}{n} - R_i$$

$$D^- = R_i - \frac{i-1}{n}$$

Step 2]

Then,

i	R_i	$\frac{i}{n}$	$\frac{(i-1)}{n}$	D^+	D^-
1	0.26	0.10	0.00	-0.16	0.26
2	0.31	0.20	0.10	-0.11	0.21
3	0.58	0.30	0.20	-0.28	0.38
4	0.61	0.40	0.30	-0.21	0.31
5	0.65	0.50	0.40	-0.15	0.25
6	0.66	0.60	0.50	-0.06	0.16
7	0.75	0.70	0.60	-0.05	0.15
8	0.88	0.80	0.70	-0.08	0.18
9	0.90	0.90	0.80	0.00	0.10
10	0.98	1.00	0.90	0.02	0.08

The maximum value,

$$D^+ = 0.02$$

$$D^- = 0.38$$

$$\therefore D = \max(D^+, D^-) = 0.38$$

Step-3] critical value, $\alpha = 0.05$
 $n = 10$

$$D_{0.05} = \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{10}} = 0.43$$

Step-4] Decision:

Since, $D = 0.38 < 0.43$

The given sequence can be considered uniformly distributed random numbers.

Q.5a] From the below given random number checks its uniformity using Chi-Square method for 95% accuracy.
(Use standard table for approximate value).

⇒ Given,

random number (n) = 100

range from 100 - 199.

total no. of class (k) = 10

Step-1] Hypothesis

H_0 : The random number are uniformly distributed.

H_1 : The random numbers are not uniformly distributed.

Step-2]. Here, $O \rightarrow$ observed frequency
 $E \rightarrow$ expected frequency.

Range	O	$E = n/k$	$(O-E)$	$(O-E)^2/E$
100-109	12	10	2	0.40
110-119	5	10	-5	2.50
120-129	10	10	0	0
130-139	18	10	8	6.40
140-149	8	10	-2	0.40
150-159	15	10	5	2.50
160-169	8	10	-2	0.40
170-179	10	10	0	0
180-189	8	10	-2	0.40
190-199	6	10	-4	1.60

$$\leq \frac{(0.1)^2}{E} = 14.2$$

$$\therefore \chi^2_{\text{cal}} = 14.2$$

Step-3] Critical value,
Degree of freedom, $df = K - 1 = 10 - 1$
 $= 9$

At 95% Confidence level

$$\chi^2_{0.05, 9} = 16.919$$

Step-4] Decision:

Since,

$$\chi^2_{\text{tab}} > \chi^2_{\text{cal}}$$

$$\therefore 16.919 > 14.2$$

Then Accept H_0 :

So, the random numbers
are uniformly distributed

5.6] A machine tool in a manufacturing shop is turning out part at a rate of one every 8 minutes. As they are finished, the parts go to an inspector, who takes 6 ± 3 minute to examine each one and rejects about 20% of the parts. The rejected parts are then sent for further refinement. Reworking takes 10 ± 3 minutes. after correction all the parts are resubmitted for inspection. Simulate for 500 part to be completed. Draw GPSS block diagram and write GPSS code.

⇒ Given

machine produces 1 part every 8 minute.

Inspection time = 6 ± 3 minutes.

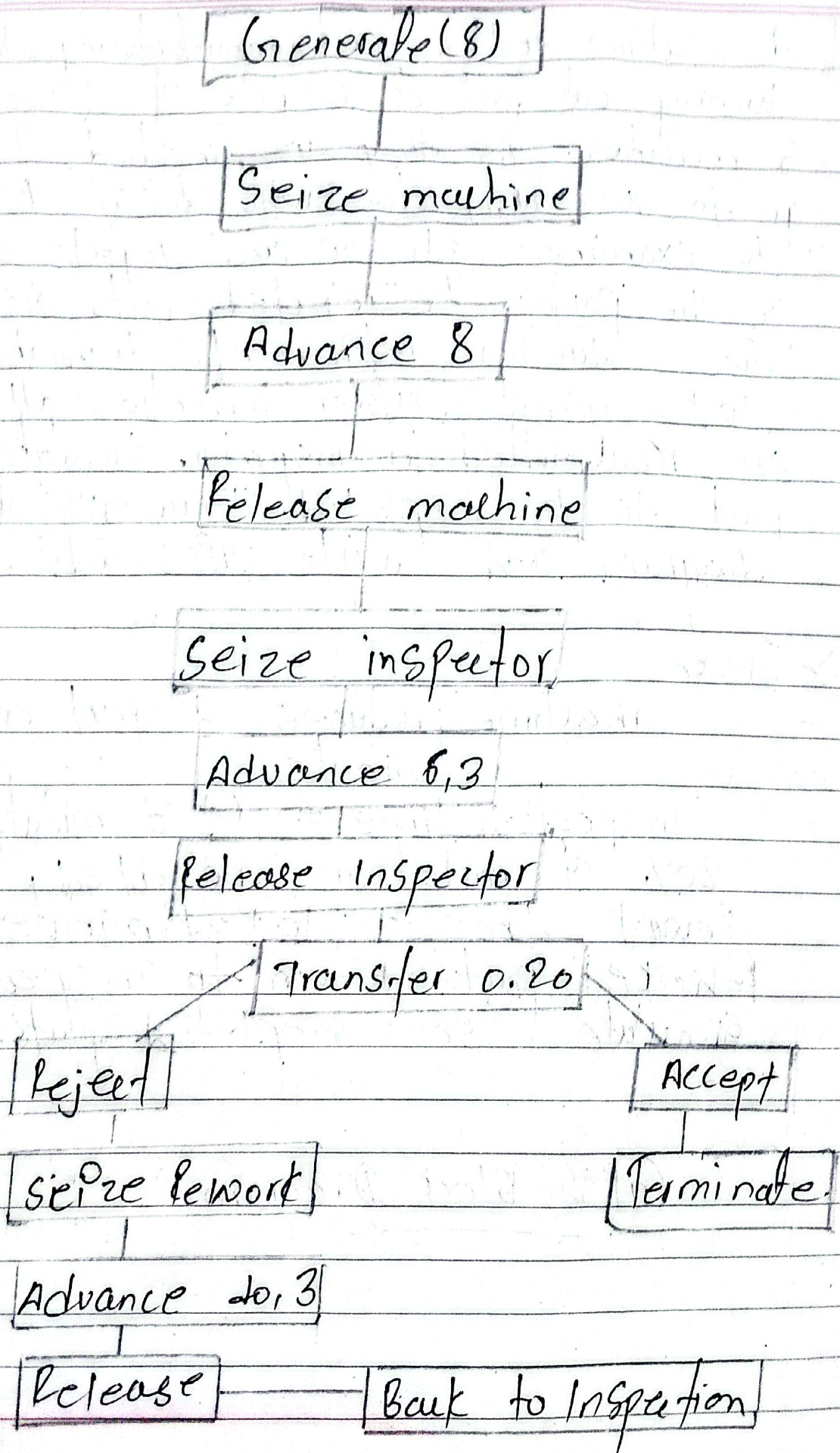
20% of parts are rejected.

Rework time = 10 ± 3 minutes.

reworked parts return to inspection

Simulate 500 completed parts.

GPSS Block Diagram:



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GPSS program :-

Simulate

Generate 8

Queue M0

Seize Machine

Depart M0

Advance 8

Release machine

Queue I0

Seize Inspector

Depart I0

Advance 6, 3

Release Inspector

Transfer 0.20, Rework, finish

Advance 10, 3

Release Rework M

Transfer, I0

Finish Terminal 1

Start 500

END

Gfss program :-

Simulate

Generate 8

Queue MQ

Seize Machine

Depart MQ

Advance 8

Release machine

Queue IQ

Seize Inspector

Depart IQ

Advance 6, 3

Release Inspector

Transfer 0.20, Rework, Finish

Advance 10, 3

Release Rework M

Transfer, IQ

Finish Terminate 1

Start 500

END

GPSS program :-

Simulate

Generate 8

Queue MQ

Seize Machine

Depart MQ

Advance 8

Release machine

Queue IQ

Seize Inspector

Depart IQ

Advance 6, 3

Release Inspector

Transfer 0.20, Rework, Finish

Advance 10, 3

Release ReworkM

Transfer, IQ

Finish Terminal 1

Start 500

END

GPSS program :-

Simulate

Generate 8

Queue MQ

Seize Machine

Depart MQ

Advance 8

Release machine

Queue IQ

Seize Inspector

Depart IQ

Advance 6, 3

Release Inspector

Transfer 0.20, Rework, Finish

Advance 10, 3

Release Rework M

Transfer, IQ

Finish Terminate 1

Start 500

END

GPSS program :-

Simulate

Generate 8

Queue MQ

Seize Machine

Depart MQ

Advance 8

Release machine

Queue IQ

Seize Inspector

Depart IQ

Advance 6, 3

Release Inspector

Transfer 0.20, Rework, Finish

Advance 10, 3

Release Rework M

Transfer, IQ

Finish Terminate 1

Start 500

END

6.a] Define SIMSCRIPT program. Explain organization of SIMSCRIPT statement.

⇒ SIMSCRIPT is a general purpose, event oriented simulation programming language developed by the RAND Corporation for modeling and simulating complex discrete event systems such as queueing systems, transportation, manufacturing, military operations and communication network.

features of SIMSCRIPT :-

- It is event oriented simulation language.
- Supports discrete event simulation.
- Uses English like syntax, making program easy to read.
- Handle the dynamic entities and queues efficiently.
- Suitable for large and complex simulation models.

#. Organization of SIMSCRIPT statements.

A SIMSCRIPT program is generally organized into the following sections.

1. Program Heading:

Contain the program title.

Example: PREAMBLE.

2. Declaration Section

Declares entities, variables, arrays, resources and attributes.

Example:

Define Customer As A Permanent Entity
Define Server As a Resource.

3. Initialization Section.

initialization the variables and schedules the first event.

Example:

let clock = 0
schedule Arrival Now

4. Event Routine section.

Define the logic for each event, such as arrival, departure or service completion.

Example:

Event Arrival
Event Departure

5. Statistics Collection.

- Collects performance measure such as
 - Waiting time
 - Queue length
 - Number of customers served etc.

6. Report Section

print the final simulation results.

Example:

Print Average waiting time
print utilization.

7. End statement

terminates the program.

Example:

END

6.6] How would you use replication of runs to calculate means, variances and confidence intervals of output parameter? Describe.

⇒ Replication of Runs

A replication means executing the same simulation model multiple times using different random number streams. Each run produces a different output because of randomness.

Replication helps obtain reliable estimate of system performance.

Steps :-

- Run the simulation n times
- Record the output from each run.
- Calculate the sample mean
- Calculate the sample variance.
- Construct a confidence interval for the true mean.

(a) Sample mean

if the output are
 $x_1, x_2, \dots, x_n$

then the Sample mean is,

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

b) Sample variance

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

This measures the variability of the simulation outputs.

c) Confidence Interval (95%):

The confidence interval for the true mean is

$$\bar{X} \pm t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$$

Where,

$\bar{X}$ = Sample mean

S = Sample standard deviation

n = number of replication

$t_{\alpha/2, n-1}$ = Student's t -value.

7. Write short notes on:

a) Elimination of Initial bias (Warm up period)

→ Initial bias is the error introduced in simulation results because the simulation starts from an artificial or empty initial state rather than the normal operating condition.

→ Why Eliminate Initial Bias

→ Improves the accuracy of simulation results.

→ Ensures the system reaches a steady state before collecting statistics.

→ Produces realistic performance measures.

→ Methods to Eliminate Initial Bias

1. → Warm-up period.

2. → Long simulation run

3. → Multiple replication.

4. → Initialization with realistic values.

→ Example: A bank simulation.

b) Generation of arrival pattern :

→ An arrival pattern describes how entities (customers, jobs, call etc) arrive at the a system over time.

→ Types of Arrival patterns:-

1. Deterministic Arrival
Arrivals occurs at fixed intervals.

2. Random (Stochastic) Arrival
Arrivals occurs randomly.

3. Batch Arrival
Several entities arrive together.

4. Time Dependent Arrival
Arrival rate change with time.

→ Importance :-

→ Helps model real world system accurately.

→ Affects queue length and waiting time.

→ Used in banks, hospitals, manufacturing and communications systems.

c) Queueing System

→ A queueing system is a system in which customer or jobs arrive, wait in a queue if service is unavailable, receive service and then leave the system.

Basic Queueing Model :-

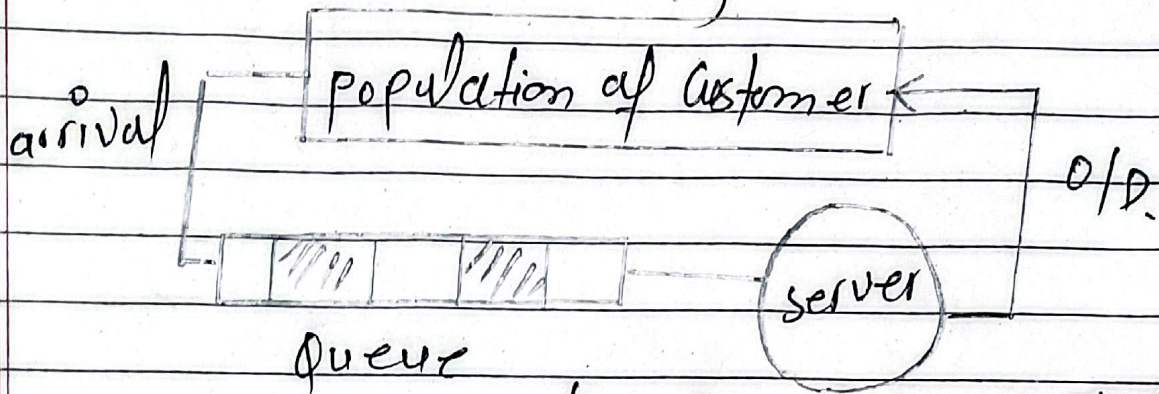


fig: Queueing Model

→ Advantages:

- Helps determine the optimum number of servers.
- Reduces waiting time
- Improve customer satisfaction
- Enhances efficient resource utilization.

→ Applications :-

Banks, Hospitals, Telephone Call Centers, Computer Network etc